"cut and paste"

$x \quad y \quad \in L(M)$

x'

M reaches q on either x or x'

$x'y$ must be $\in L(M)$ too

Those who cannot remember the past are condemned to repeat it.

-- George Santayana (1905) Life of Reason

Corollary

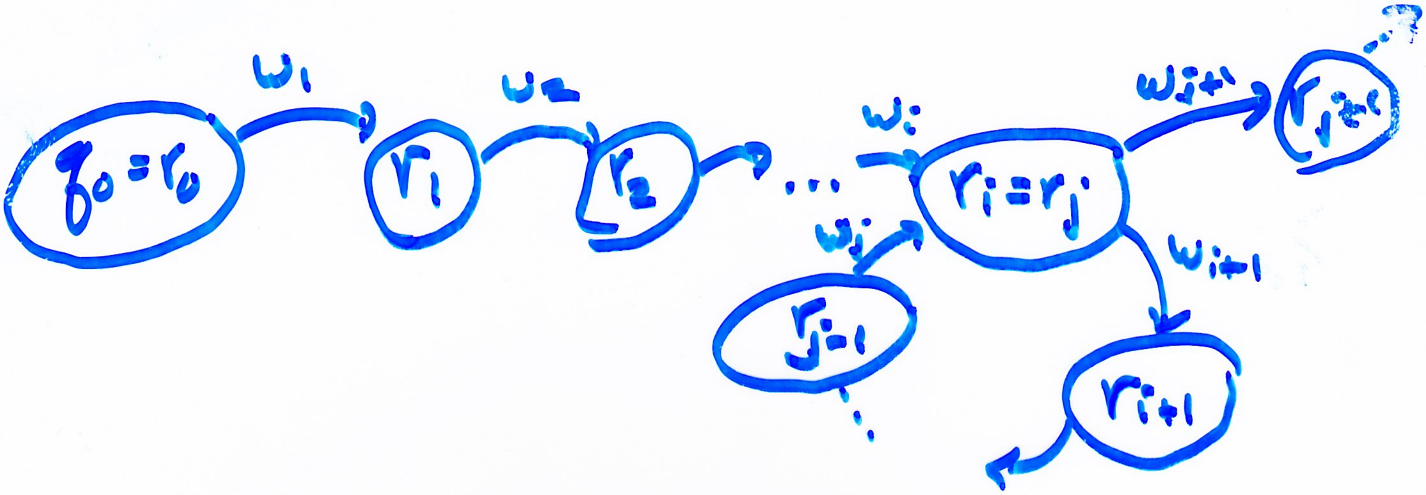
Every sufficiently long input string forces a DFA around a loop.

Proof

Let $p = |Q|$ and $|w| \geq p$.

Let r_i, $0 \leq i \leq |w|$ be state M's in after reading 1st i letter of w.

By pigeonhole principle $\exists\, 0 \leq i < j \leq |w|$ st $r_i = r_j$.


$q_0 = r_0$
w_1
r_1
w_2
r_2
w_i
$r_i = r_j$
w_{j+1}
r_{j+1}
w_j
r_{j-1}
w_{i+1}
r_{i+1}

Pumping Lemma

$\forall$ regular language L

$\exists p \quad \forall w \in L \quad |w| \geq p \Rightarrow$

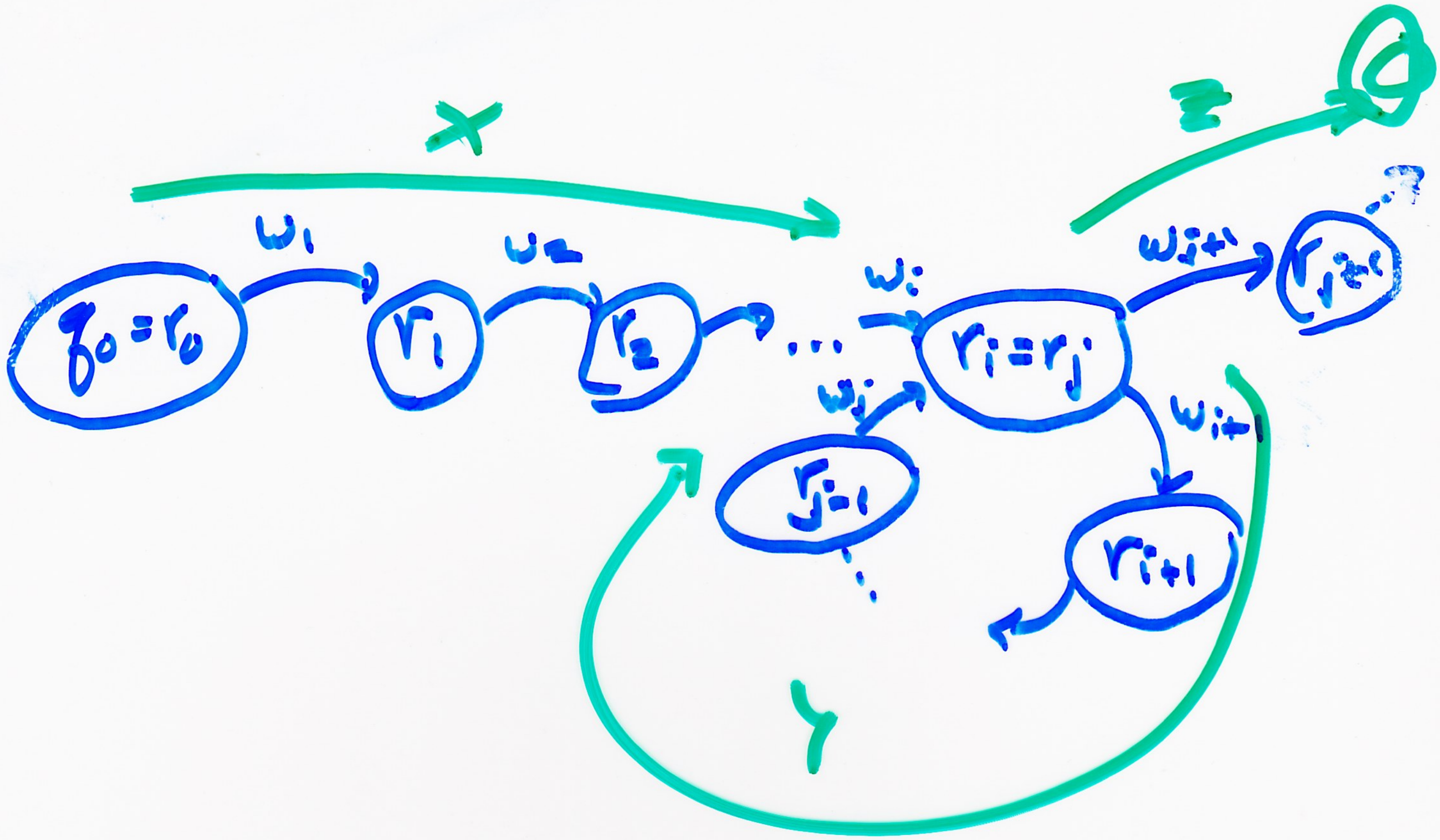
$\exists x, y, z \in \Sigma^*$ s.t.

$w = xyz$

$y \neq \varepsilon$

$|xy| \leq p$

$\forall i \geq 0 \quad xy^iz \in L$



16-5

16-3

P.L. suggests all regular languages are infinite!?? Surely false...

Eg. Suppose $L = \{a\}$

PL says $\exists p \ \forall w \in L \ |w| \geq p \Rightarrow \ldots$

Well, take $p = 2$. Then, yes indeed for all strings in L of length 2 or greater $\exists\, x y z \ldots$ is vacuously true, since there are no such strings in L.

Ditto for any finite language — $p = 1 +$ max length string in L.

$L = \{ a^n b^n \mid n \geq 0 \}$

if L is regular then by P.L.

$\exists p$ st ...

$w = a^p b^p$

$\exists\ x, y, z \in \Sigma^*$

st $xyz = w$

$|y| > 0$

$|xy| \leq p$

$x = a^i$ for some $0 \leq i < p$

$y = a^j$ for som $1 \leq j \leq p$

$z = a^{p-i-j} b^p$

$xy^2z = a^{p+j} b^p \notin L$

$\therefore$ L is not regular.