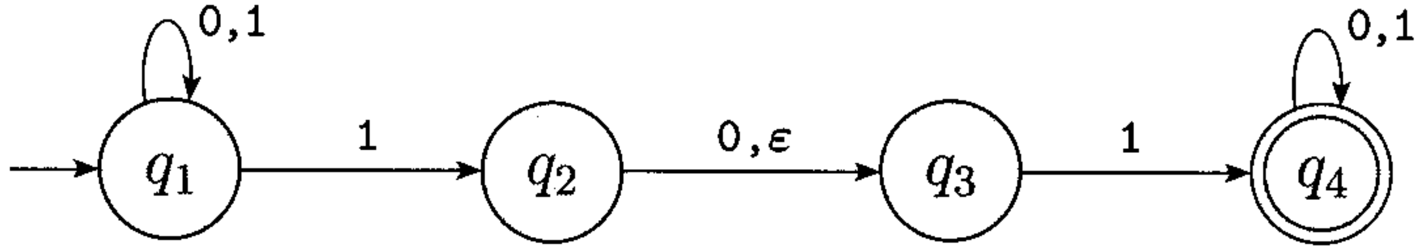
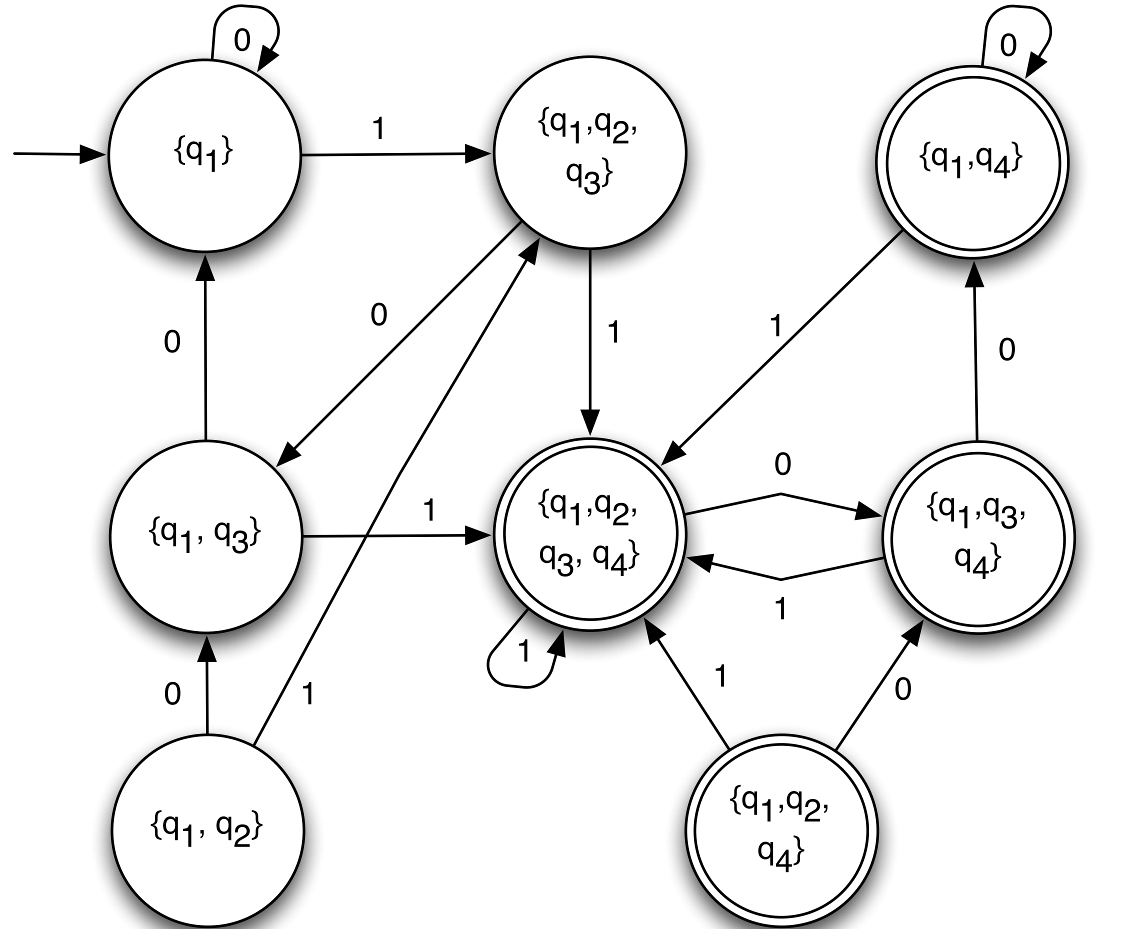
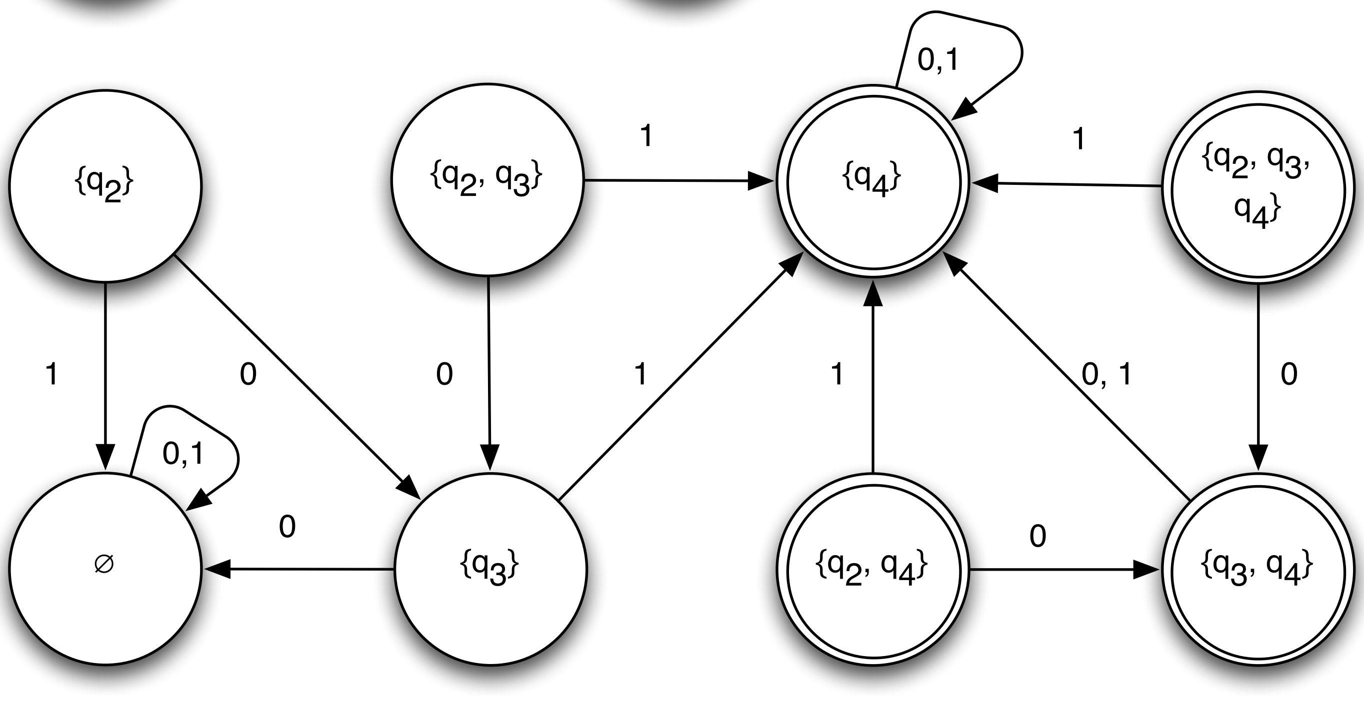


FIGURE 1.27



Notes on Subset Construction:
1) only the top 6 states are reachable from the start state, but all 16 are required by the construction.
2) ε moves come *after* Σ moves. E.g.,
$\delta'(\{q_2\},1) = \varnothing$,
not $\{q_4\}$.



Defn

M_1 & M_2 **equivalent** if $L(M_1) = L(M_2)$

Theorem 1.39

$\forall$ nfa N $\exists$ equivalent dfa M

given $N = (Q, \Sigma, \delta, q_0, F)$

build $M = (Q', \Sigma, \delta', q_0', F')$

~~(warm up: no ε-moves)~~ $\rightarrow$ Full version: with ε-moves

$$Q' = 2^Q$$

$$q_0' = E(\{q_0\})$$

$$F' = \{R \subseteq Q \mid R \cap F \neq \emptyset\}$$

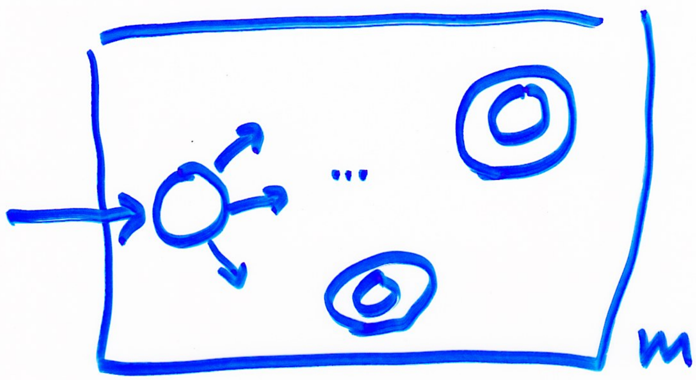
$\forall a \in \Sigma$, $\forall R \subseteq Q$:

$$\delta'(R, a) = \bigcup_{r \in R} E\Big(\delta(r, a)\Big)$$

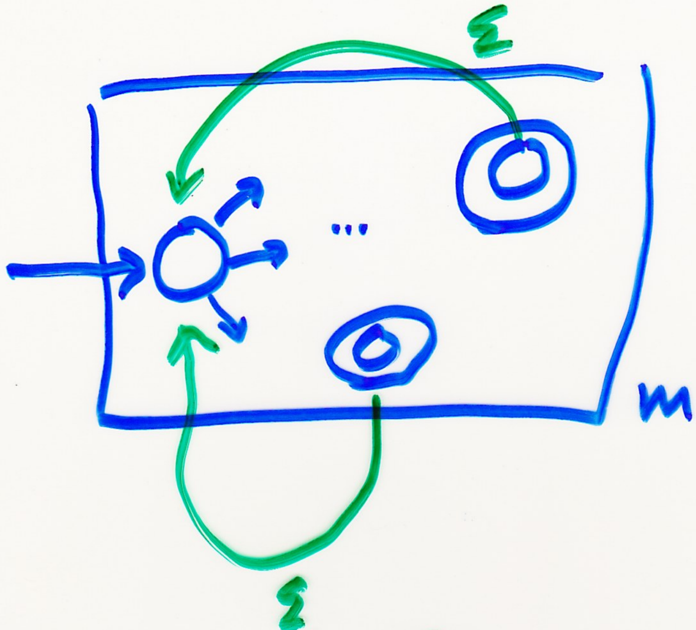
$\forall R \subseteq Q$

$E(R) = \{q \mid q$ reachable by 0 or more ε-moves from some $r \in R\}$

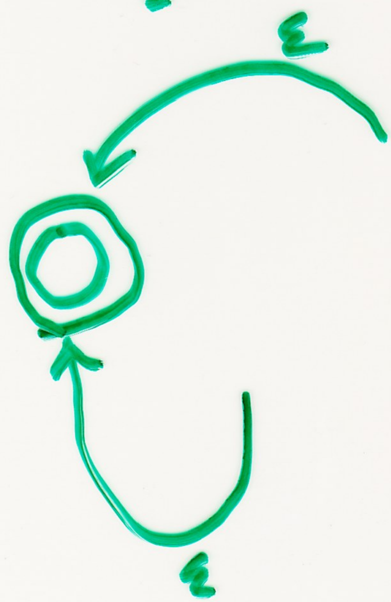
Given NFA M, can build one for $L(M)^*$?



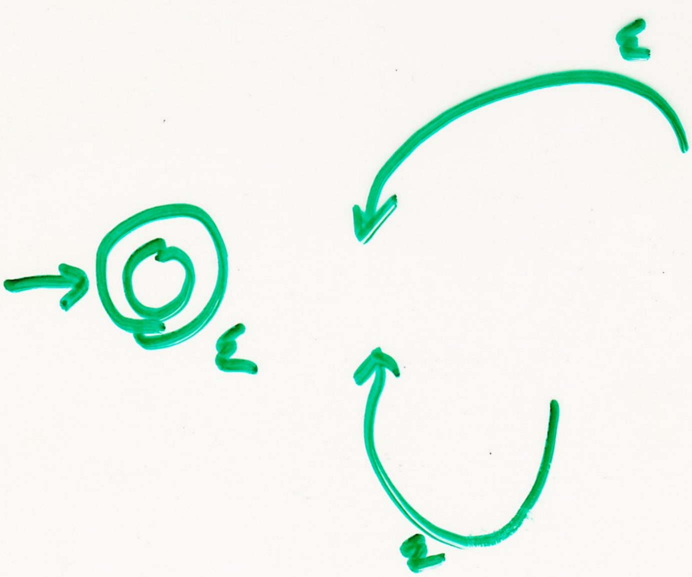
Given NFA M, can build one for $L(M)^*$?



No
(may reject ε)

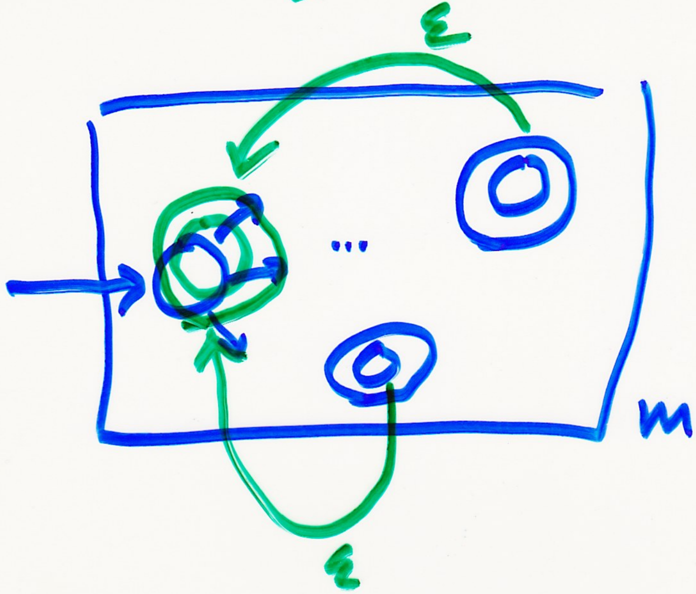


No
May accept
extra
stuff

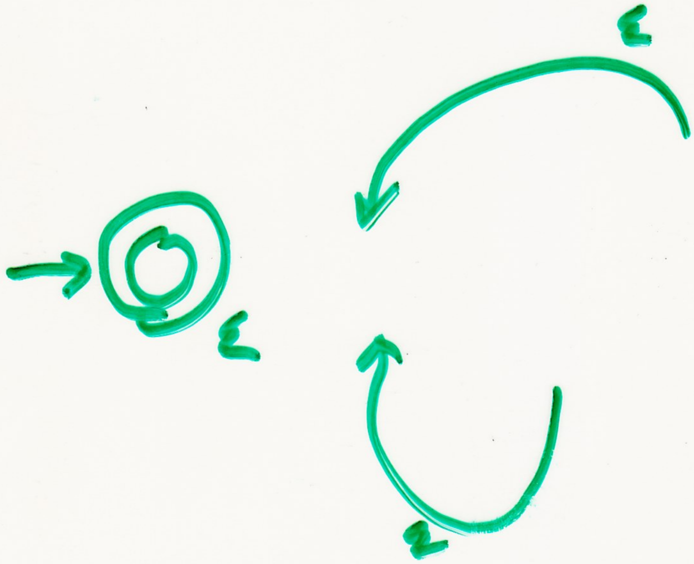


Yes!

Given NFA M, can build one for $L(M)^*$?



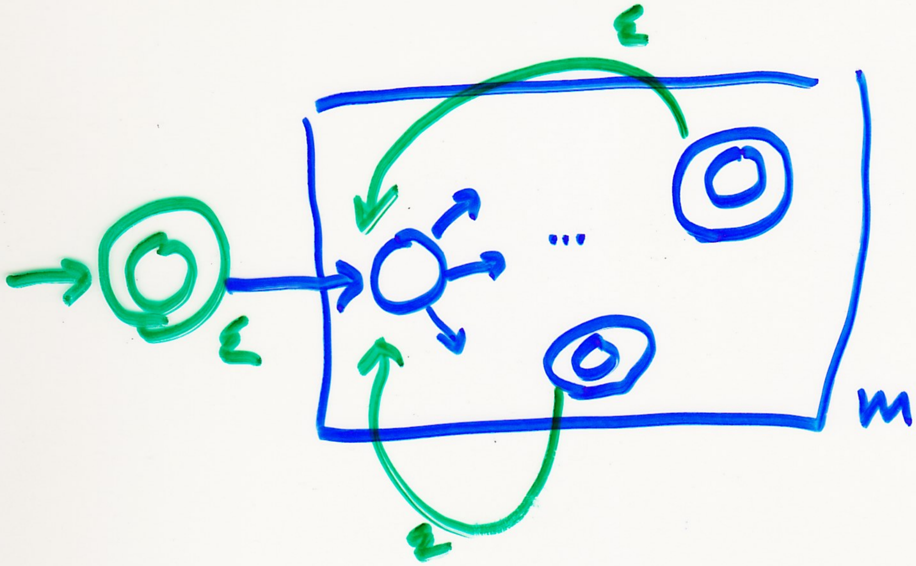
No
May accept
extra
stuff



Yes!

stuff

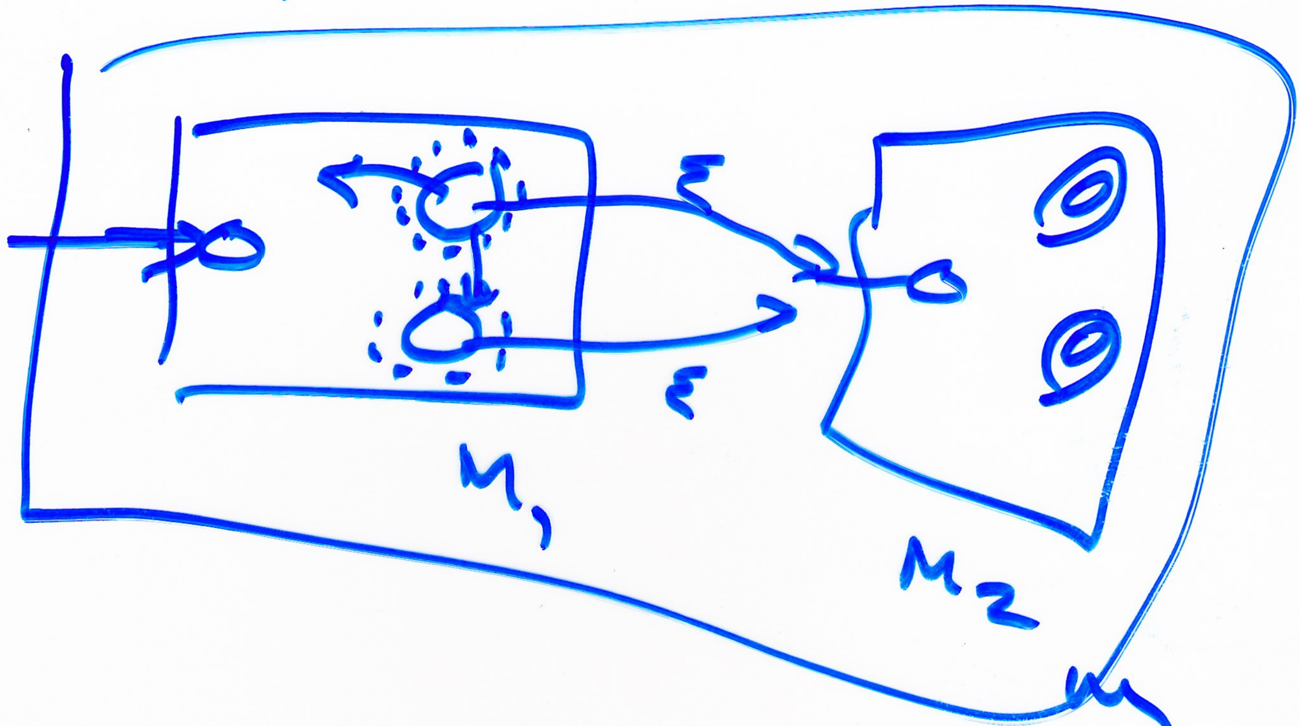
Given NFA M, can build one for $L(M)^*$?



Yes!

2 NFA's M_1 M_2 $L_i := L(M_i)$

$L_1 \cdot L_2$



I. Suppose $x \in L_1$, $y \in L_2$

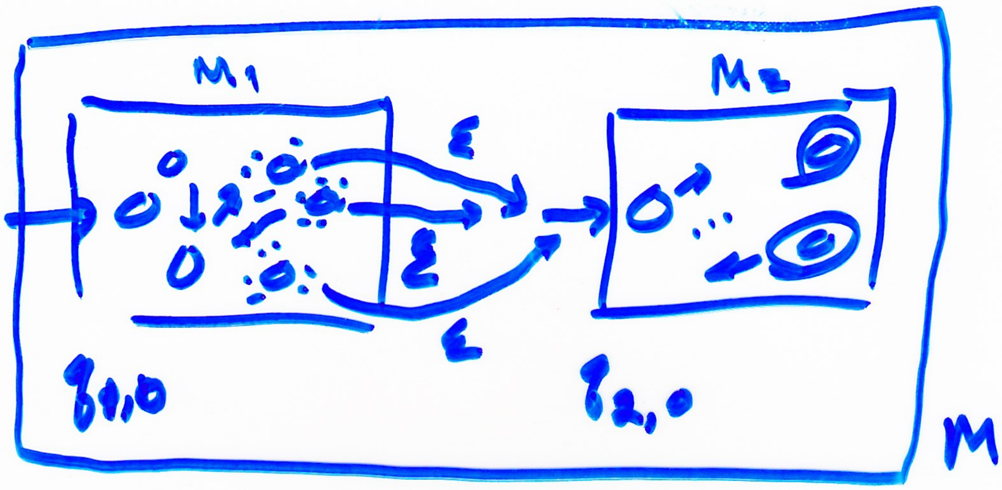
Then M_1 reading x can reach a final state, say q. By construction,

$q_{20} \in \delta(q, \varepsilon)$ (where q_{20} = init of M_2)

And from $q_{2,0}$ reading y, M_2 reaches a final state. $\therefore$ M reading xy can reach a final state so xy accepted by M

$\therefore$ $\boxed{L_1 \cdot L_2 \subseteq L(M)}$

For $i = 1, 2$, NFA M_i, $L_i = L(M_i)$



I. if $x \in L_1$, $y \in L_2$ then $xy \in L(M)$ ...

II. Suppose $w \in L(M)$

So M reaches F reading w.

But no state of M_1 is in F and only transitions between M_1 & M_2 are ε-transitions <u>from</u> F_1 <u>to</u> q_{20}

So, reading w, M stays in M_1 a while (reading some prefix of w, call it x) then jumps from some $q \in F_1$ to q_{20}, then runs around in M_2 reading rest of w (call it y) ending in $F_2 = F$.

$\therefore$ $w = xy$ s.t. $x \in L_1$ & $y \in L_2$

$$\boxed{L(M) \subseteq L_1 \circ L_2}$$

Regular expressions over Σ

$\emptyset$ is an r.e.

ε . . . . r.e.

a . . . . . for each $a \in \Sigma$

if R_1 & R_2 are r.e.s,
then so are

$(R_1 \cup R_2)$

$(R_1 \circ R_2)$

$(R_1{}^*)$

---

the language denoted by R, $L(R)$
is :

$L(\emptyset) = \emptyset$

$L(\varepsilon) = \{\varepsilon\}$

$\vdots$

$L(R_1 \cup R_2) = L(R_1) \cup L(R_2)$

$$L(\underbrace{(\phi^*)}_{\text{R.E.}}) = L(\phi)^*$$

$$= \phi^*$$

$$= \{\varepsilon\}$$

Short hands

$$\Sigma = \{a, b, c\}$$

$$L(((a \cup b) \cup c)) = \Sigma$$

$$(\underline{\Sigma}^* \cup \varepsilon) \cdot a$$

$$(((\underline{(a \cup b) \cup c})^* \cup \varepsilon) \cdot a)$$

precedence & associativity

$$(a \cup b \cup c)$$

$$a \cup b \cdot c^*$$

$$(a \cup (b \cdot (c^*)))$$

"words ending with ".TXT" "

$\Sigma^* . TXT$

$(a \cup b \cup \dots \cup Z) \cdot (a \cup \dots \cup Z \cup 0 \dots 9)^*$

$l \cdot (l \cup d)^*$

Shorthand

$(\Sigma\Sigma)^*$

$\left.\begin{array}{l}\Sigma\Sigma^* \\ aa^*\end{array}\right\}\begin{array}{l}\Sigma^+ \\ a^+\end{array}$

$0^* 1 0^*$

$(\varepsilon \cup \Sigma)(\varepsilon \cup \Sigma)$

$\Sigma\Sigma$

$00 \in \quad 0^*(10^*10^*)^*$

$00 \notin \quad (0^*10^*10^*)^*$

$(0^*10^*1)^*0^*$

$(d^*.d^+ \cup d^+.d^*)(\varepsilon \cup E(\varepsilon \cup + \cup -)d^+)$

9-3