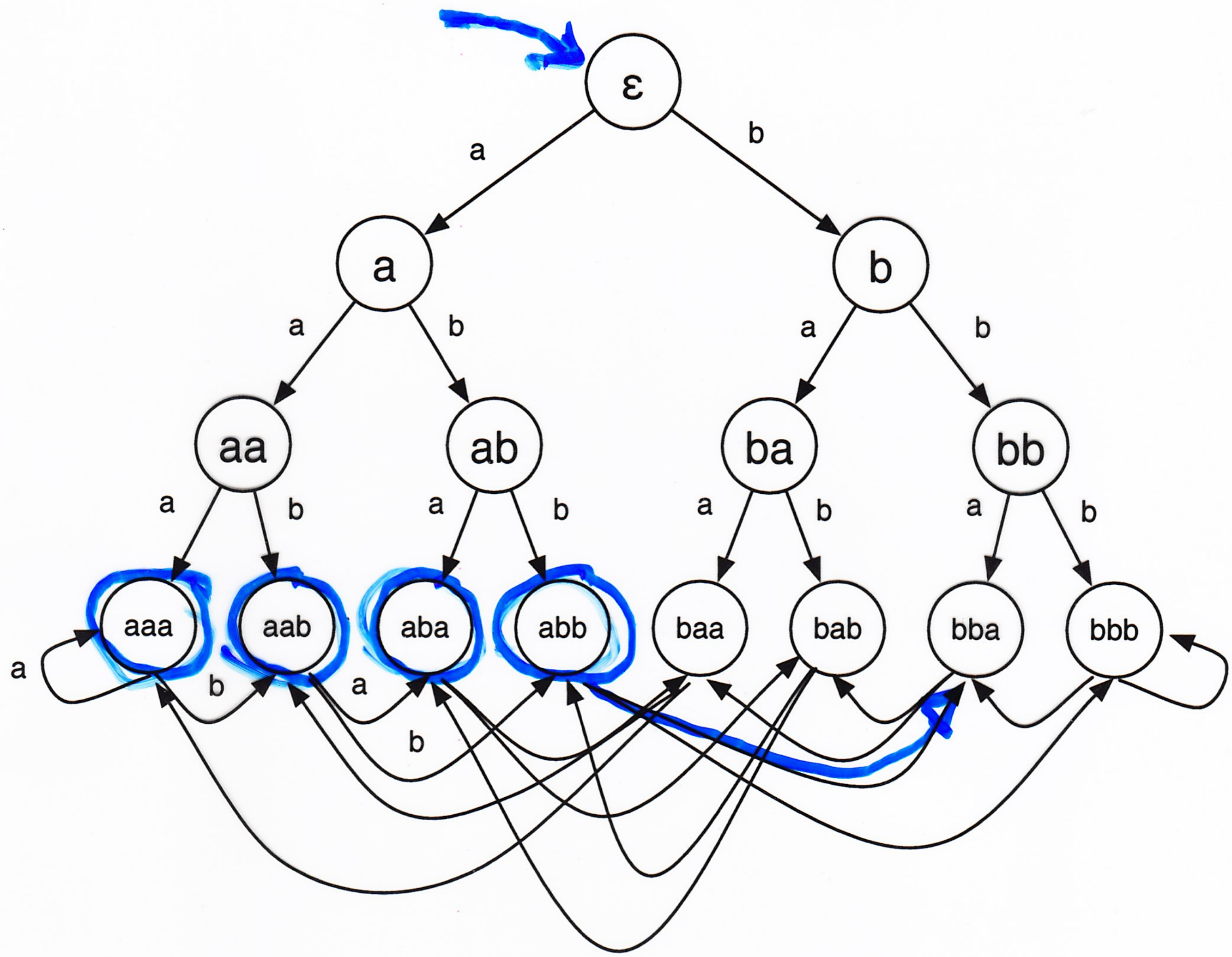
L = { w in {a,b}* | 3rd letter from the right end of w is "a" }



$M = (Q, \Sigma, \delta, q_0, F)$

$\Sigma = \{a, b\}$

$Q = \{ w \in \Sigma^* \mid |w| \leq 3 \}$

$q_0 = \varepsilon$

$F = \{ w \in \Sigma^* \mid w = ax, |x| = 2 \}$

$\forall w \in Q$
$\forall c \in \Sigma$ $\delta(w, c) =$ Last 3 letters of $w.c$

~~"shift register"~~

DEFN ("is in state q")

M ends in state q after reading $w \in \Sigma^*$ if

(1) $w = w_1 w_2 \cdots w_n$

where $w_i \in \Sigma$

(2) $\exists$ state $r_0, r_1, r_2 \cdots r_n \in Q$

s.t. (a) $r_0 = q_0$

(b) $\forall\, 1 \le i \le n$

$$\delta(r_{i-1}, w_i) = r_i$$

(c) $r_n = q$

Fact: q is unique

because δ is a function, basically

Defn

M accepts $w \in \Sigma^*$ $\Leftrightarrow$ the state, q, reached by M after reading w is an accepting state, i.e., $q \in F$.

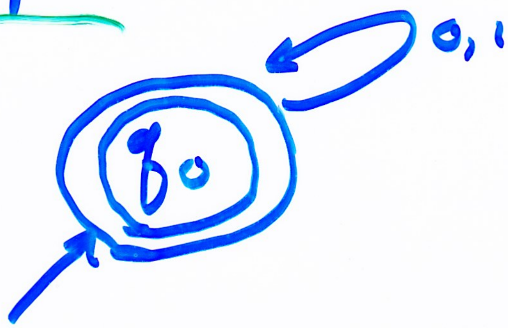
Defn

The language recognized by M, $L(M) = \{ w \in \Sigma^* \mid M \text{ accepts } w \}$.

Note

Every M recognizes exactly one language. Implicitly, it "recognizes" both strings it must accept and those it must reject.

Example

M: 

$L(M) = \Sigma^*$

$L_{pal} = \{ w \in \{0,1\}^* \mid w = w^R \}$

e.g. 101 and 001100 are palindromes
110 is not

M above accepts every palindrome

$\therefore \; L_{pal} \subseteq L(M)$

but M also accepts some (in fact, all) non palindromes

$\therefore \; L_{pal} \neq L(M)$

Regular Languages

$L \subseteq \Sigma^*$ is regular iff
$L = L(M)$ for some F.A. M

Examples

"even parity" is regular
"3rd from right" is regular
"odd length" is regular
"Σ^*" is regular

Are there general ways to prove languages are regular, other than making more & more example M's?

Theorem

If L is regular then so is $\Sigma^* - L$

Proof

L regular, so $L = L(M)$ for some fa $M = (Q, \Sigma, \delta, q_0, F)$

Let $M' = (Q, \Sigma, \delta, q_0, Q - F)$

For all $w \in \Sigma^*$:

M accepts w $\Leftrightarrow$

M is in a state $q \in F$ after reading w

$\Leftrightarrow$ M' .. ..

$\Leftrightarrow$ M' rejects w (since $q \in F \Leftrightarrow q \notin Q - F$)

$\therefore$ $w \in L(M) \Leftrightarrow w \notin L(M')$

i.e. $L(M') = \Sigma^* - L$ is regular.

Closure Properties

A set is "closed" under some operation if applying the op to set members always yeilds a set member

Examples

$\mathbb{N}$ is closed under $+$ $\times$ (eg $1+2 \in \mathbb{N}$)

but not under $-$ $/$ (eg $1-2 \notin \mathbb{N}$)

$\mathbb{Z}$ is closed under $+$ $-$ $\times$ ($1-2 \in \mathbb{Z}$)

but not under $/$ ($1/2 \notin \mathbb{Z}$)

The set of regular languages is closed under complementation