CSE 322

Intro to Formal Models in CS

Homework #6

Due: Friday, 19 Feb 10

12 Feb 10

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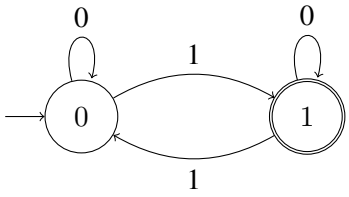
Again three separate, stapled, turn-in bundles, with your name on each, please: Problem(s) 1–3 in one, problem(s) 4 in another and problem(s) 5 in the third. Text problems below are on pages 128-132 of Sipser, *US second edition*; see online scanned versions if you don't have it.

1. 2.1. Give only *leftmost* derivations.
2. 2.4(b, c, e, f).
3. 2.6(d).
4. In homework 2, for the DFA M below,



you proved for all $i \in Q$ and $w \in \Sigma^*$ that M is in state i after reading w if and only if $\#_1(w) \equiv i \pmod 2$, where $\#_1(w)$ is the number of 1's in the string w.

The following context-free grammar is closely related: $G = (V, \Sigma, R, S_0)$, where $V = \{S_0, S_1\}$ and R is the set of rules:

$$\begin{aligned} S_0 &\rightarrow 0S_0 \mid 1S_1 \\ S_1 &\rightarrow 0S_1 \mid 1S_0 \mid \varepsilon \end{aligned}$$

   (a) List the sequence of states visited by M while accepting the string 001011.

   (b) Give a derivation of that string in G.

   (c) Prove, for all $S_i \in V$ and $w \in \Sigma^*$ that $S_0 \Rightarrow^* wS_i$ if and only if M is in state i after reading w.

   (d) Use this to prove that $L(G) = L(M)$.

   (e) Extra Credit. Generalize this example to show that for every DFA M there is a context-free grammar G such that $L(G) = L(M)$. I.e., every regular language is a context-free language. (Cor. 2.32 in the text proves the same fact in a very different way.)

5. Let L be a regular language and p the number of states in some DFA recognizing L. Prove that L is infinite if and only if there is some $x \in L$ with $p \leq |x| < 2p$.