

CSE 322
Intro to Formal Models in CS
Homework #5
Due: Friday, 12 Feb 10
5 Feb 10

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Again three separate, stapled, turn-in bundles, with your name on each please: Problem(s) 1 in one, problem(s) 2–3 in another and problem(s) 4–6 in the third. Text problems below are on pages 83-93 of Sipser, *US second edition*; see online scanned versions if you don't have it.

1. [30 pts] Let $\Sigma = \{0, 1, \#\}$, and $L = \{x\#y \mid x, y \in \{0, 1\}^* \text{ and when interpreted as binary numerals, } y \text{ is the square of } x\}$. E.g., 011#1001 is in L ($3^2 = 9$), but 1#10 is not. In this problem you will give *three* proofs that L is not regular.
 - (a) Use a cut-and-paste style proof as on slide 15-3 of the lecture notes, involving many equal-length strings. Please note that I would give myself a grade of about 4/10 if I turned in that slide as a solution to a problem like this, with the note “right idea, but way too terse; you need to justify many steps much more fully.” (Hopefully, a transcript of my verbal justification would score higher, but the TA's, unlike your charming professor, are SO MEAN, that you'd best be careful. Ditto for all following problems ...)
 - (b) Repeat the proof using the “increasingly long simple strings” style found on slide 15-5. Again, justify the various steps more carefully than I did on the slides...
 - (c) Prove it again using the Pumping Lemma (Thm 1.70). Note that a proof based on the pumping lemma should make *no mention* of DFAs or states, unlike the methods in the previous two parts.
2. Show that $L = \{www \mid w \in \{a, b\}^*\}$ is not regular. In this and subsequent problems in this assignment, you may use any sound method, including any of the three methods outlined above.
3. Let $\Sigma = \{a, b\}$.
 - (a) Prove that $G = \{w \in \Sigma^* \mid w \text{ is a palindrome}\}$ is not regular.
 - (b) Prove that $F = \{w \in \Sigma^* \mid w \text{ is not a palindrome}\}$ is not regular. [Hint: see exercise 1.14 (1st ed.: 1.10).]
4. 1.30 (1st ed.: 1.18)
5. For any two strings $x, y \in \Sigma^*$ of equal length, define the *Hamming distance* between them $H(x, y)$ to be the number of indices i such that the i^{th} letters of x and y disagree. E.g., $H(x, y) = 0 \Leftrightarrow x = y$, and for any x there are $|x| * (|\Sigma| - 1)$ strings y such that $H(x, y) = 1$; i.e., y disagrees with x in any one of $|x|$ positions. Let $L = \{xy \mid H(x, y) = 1, \text{ where } |x| = |y|, x, y \in \Sigma^*, |\Sigma| \geq 2\}$. Show that L is not regular.
6. Extra Credit: 1.54 (1st ed.: not present)