

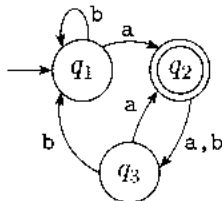
CSE 322
Intro to Formal Models in CS
Homework #1
Due: Friday 8 Jan 10

W. L. Ruzzo

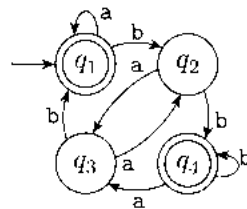
4 Jan 10

Read the section on “Collaboration” on the course home page. Turn in your solutions on paper in class. If you can’t attend, bring the paper to Ruzzo’s office before class, or submit a copy electronically (e.g., a scanned version) using the Catalyst drop box linked from the course home page. FAX is OK if you label it clearly as destined for Ruzzo and send email to let him know to look. In all cases, homework is due by the beginning of class on the due date.

1. Let A be the set $\{2, 3, 5\}$ and $B = \{2, 5\}$
 - (a) Is A a subset of B ?
 - (b) Is B a subset of A ?
 - (c) What is $A \cup B$?
 - (d) What is $A \cap B$?
 - (e) What is $A \times B$?
 - (f) What is the powerset of B ?
2. If A is a set with n elements, how many elements are in the powerset of A ? Explain your answer.
Extra Credit: prove it by induction.
3. Consider the undirected graph $G = (V, E)$ where the set of nodes is $V = \{1, 2, 3, 4\}$ and the set of edges is $E = \{\{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 4\}, \{2, 4\}\}$. Draw the graph G . What is the degree of node 1? of node 3? Indicate a path from node 3 to node 4 in your drawing.
4. Below are state diagrams of two DFAs, M_1 and M_2 .



M_1



M_2

- (a) What is the start state of M_1 ?
- (b) What is the set of accept states of M_1 ?
- (c) What is the start state of M_2 ?
- (d) What is the set of accept states of M_2 ?
- (e) What sequence of states does M_1 go through on input $aabb$?

- (f) Does M_1 accept the string $aabb$? Explain why or why not.
- (g) Does M_2 accept the string ε ? Explain why or why not.
5. Give state diagrams for DFAs recognizing the following languages. The alphabet is $\Sigma = \{0, 1\}$ in all cases.
- (a) $\{w \mid w \text{ begins with a 1 and ends with a 0}\}$.
- (b) $\{w \mid w \text{ contains at least three 1s}\}$.
- (c) $\{w \mid \exists x, y \in \Sigma^* \text{ s.t. } w = x0101y, \text{ i.e., } w \text{ contains the substring } 0101\}$.
- (d) $\{w \mid \text{the length of } w \text{ is at most } 5\}$.
- (e) $\{w \mid \text{every odd position of } w \text{ is } 1\}$. (The first, third, . . . letters of a string are odd positions; i.e., 1-based indexing, unlike the zero-based indexing common in programming languages.)
- (f) The empty set.