CSE 322: The Last Homework Assignment

Due Date: Friday, June 4 (at the *beginning* of class)

1. (30 points) Show that the language SEARCH = {$w\#x$ | w, $x \in \{0,1\}^*$ and w is a substring of x} is <u>decidable</u> by giving an *implementation level description* of a decider TM M for the language. (Give your TM in the format M = "On input string *s*:…"; no need to give the state diagram). See examples of implementation level descriptions in Section 3.1 in the text. You may use a deterministic or nondeterministic TM with a single tape or multiple tapes.

2. (40 points: 10 each)
   i. Show that decidable languages are closed under complement.
   ii. Show that decidable languages are closed under concatenation.

   Give *implementation level details* of the necessary Turing machines in each case.

   iii. Can you modify your proof of (i) above to show that Turing-recognizable languages are closed under complement? Explain how. If not possible, explain why not.
   iv. Can you modify your proof of (ii) above to show that Turing-recognizable languages are closed under concatenation? Explain how. If not possible, explain why not.

3. (30 points) Let $NO\text{-}INT_{TM} = \{ \langle A,B\rangle \mid A \text{ and } B \text{ are TMs and } L(A) \cap L(B) = \emptyset\}$. Show that $NO\text{-}INT_{TM}$ is <u>undecidable</u> by giving a reduction from a known undecidable language to $NO\text{-}INT_{TM}$. For your reduction, you may use any of the languages shown to be undecidable in Section 5.1 in the textbook (up to Theorem 5.4 and its proof; no need to read beyond this theorem for this course). (Hint: Use a proof similar to the one for Theorem 5.4 in the textbook.)

Just for fun (no points): Is the question "Does God exist?" decidable? (Hint: Assume the question has an unambiguous yes or no answer).