



The Last-but-one Homework Assignment

Due Date: Friday, May 28 (at the *beginning* of class)

1. (25 points) Convert the following CFG G over $\Sigma = \{a, b\}$ to an equivalent PDA using the procedure discussed in class (Lemma 2.21 in the text):
$$S \rightarrow aSb \mid bY \mid Ya$$
$$Y \rightarrow bY \mid aY \mid \varepsilon$$

Show all states in your state diagram for the PDA, including the extra states used for pushing strings onto the stack (see, for example, the state diagram in Example 2.25 in the text).
2. (25 points; 10 for part a and 15 for part b) For any two regular languages A and B , define the language $L = \{xy \mid x \in A, y \in B, \text{ and } |x| = |y|\}$.
 - a. Show that L need not be regular by giving an example of A , B , and L .
 - b. Show that L is a context free language by giving a detailed but informal description of a PDA that accepts L . See Example 2.18 in the textbook for the level of detail required for the description. You do not need to draw the state diagram. (Hint: Make use of DFAs for A and B in constructing your PDA and recall the construction for showing regular languages are closed under concatenation).
3. (30 points) Use the pumping lemma to show that the following languages are not context free:
 - a. $\{a^n b^{2^n} a^n \mid n \geq 0\}$ over $\Sigma = \{a, b\}$
 - b. $\{w \mid w \in \{a,b,c,d\}^* \text{ and the number of } a\text{'s in } w \text{ is equal to the number of } b\text{'s and the number of } c\text{'s in } w \text{ is equal to the number of } d\text{'s}\}$
4. (20 points) Give the sequence of configurations that each of the following Turing machines enters when started on the indicated input strings:
 - a. Turing machine M_2 from Example 3.7 in the text on:
 - (i) input string 0
 - (ii) input string 000
 - b. Turing machine M_1 from Example 3.9 in the text on:
 - (i) input string 0#1
 - (ii) input string 01#01