## CSE 322 Spring 2010

## Homework Assignment \#5

Due Date: Friday, May 21 (at the beginning of class)

1. (16 points) Consider the CFG $G_{2}$ on page 101 in the textbook. For each of the following strings, state whether or not the string is in $L\left(G_{2}\right)$. If it is not in $L\left(G_{2}\right)$, explain why. If it is in $\mathrm{L}\left(G_{2}\right)$, give a parse tree and a leftmost derivation for the string (see page 106 for a definition of leftmost derivation):
a. the boy touches the flower
b. girl sees boy
c. the girl touches with a flower
d. the boy sees the girl with a girl
2. (20 points) Problem 2.27 in the textbook, both parts.
3. (20 points) Show that context-free languages are closed under the regular operations as well as the reversal operator, i.e., if $L_{1}$ and $L_{2}$ are any two contextfree languages such that $L_{1}=L\left(G_{1}\right)$ and $L_{2}=L\left(G_{2}\right)$ for two CFGs $G_{1}$ and $G_{2}$, show that the following languages are also context-free:
a. $\mathrm{L}_{1} \cup \mathrm{~L}_{2}$
b. $\mathrm{L}_{1}{ }^{\circ} \mathrm{L}_{2}$ ("o" denotes concatenation)
c. $\mathrm{L}_{1}$ *
d. $\mathrm{L}_{1}{ }^{\mathrm{R}}$ ("R" denotes string reversal)
4. (24 points) Give CFGs that generate the following languages:
a. $\quad\left\{w \in\{0,1\}^{*} \mid w\right.$ starts with 1 and has odd length $\}$
b. $\left\{0^{\mathrm{i}} 10^{\mathrm{j}} 10^{\mathrm{k}} \mid \mathrm{i}=\mathrm{j}\right.$ or $\mathrm{i}=\mathrm{k}$ for $\left.\mathrm{i}, \mathrm{j}, \mathrm{k} \geq 0\right\}$
c. $\left\{w \in\{0,1\}^{*} \mid\right.$ the number of 0 s in $w$ is two times the number of 1 s in $\left.\left.w\right\}\right\}$

Try this for practice but you don't need to turn it in:
The complement of the language $\left\{0^{\mathrm{n}} 1^{\mathrm{n}} \mid \mathrm{n} \geq 0\right\}$
5. (20 points) Give informal descriptions and state diagrams of pushdown automata (PDA) for the languages in 4 (b) and 4 (c) above (see Example 2.18 in the text for an example of an informal description). Note: Do not use the construction for converting CFGs to PDAs (Lemma 2.21 in the text) for this exercise - construct the PDA directly by thinking of an algorithm for accepting the language.

