CSE 322

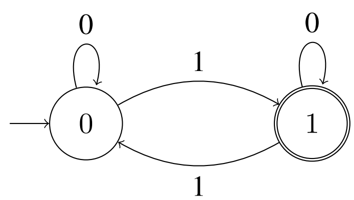
Intro to Formal Models in CS
Homework #6
Due: Friday, 19 Nov 10

12 Nov 10

W. L. Ruzzo

Again three separate, stapled, turn-in bundles, please: Problems 1–2 in one, problems 3–4 in another and problems 5–6 in the third. Text problems below are on pages 128-132 of Sipser, *US second edition*; see online scanned versions if you don't have it.

1. Let L be a regular language and p the number of states in some DFA recognizing L. Prove that L is infinite if and only if there is some $x \in L$ with $p \leq |x| < 2p$.
2. 2.1. Give only *leftmost* derivations.
3. 2.4(b, c, e, f). Also do 2.5 for part e, i.e., give a PDA for the same language and informally explain why it is correct. (You may, but need not, follow one of the CFG-to-PDA constructions; if you do, say which.)
4. 2.6(d).
5. For the DFA M below,



   for all $i \in Q$ and $w \in \Sigma^*$, M is in state i after reading w if and only if $\#_1(w) \equiv i \pmod 2$, where $\#_1(w)$ is the number of 1's in the string w. (You showed a similar result in problem #1 of homework 2.)

   The following context-free grammar is closely related: $G = (V, \Sigma, R, S_0)$, where $V = \{S_0, S_1\}$ and R is the set of rules:

$$\begin{aligned} S_0 &\rightarrow 0S_0 \mid 1S_1 \\ S_1 &\rightarrow 0S_1 \mid 1S_0 \mid \varepsilon \end{aligned}$$

   (a) List the sequence of states visited by M while accepting the string 001011.
   (b) Give a derivation of that string in G.
   (c) Prove, for all $S_i \in V$ and $w \in \Sigma^*$ that $S_0 \Rightarrow^* wS_i$ if and only if M is in state i after reading w.
   (d) Use this to prove that $L(G) = L(M)$.
   (e) Extra Credit. Generalize this example to show that for every DFA M there is a context-free grammar G such that $L(G) = L(M)$. I.e., every regular language is a context-free language. (Cor. 2.32 in the text proves the same fact in a very different way.)
6. 2.16. Just do *; I did concatenation in lecture.