CSE 322

Intro to Formal Models in CS

Homework #5 (Rev. B)

Due: Friday, 12 Nov 10

6 Nov 10

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Again three separate, stapled, turn-in bundles, with your name on each please: Problem 1 in one, problem 2 in another and problems 3–7 in the third. Text problems below are on pages 83-93 of Sipser, *US second edition*; see online scanned versions if you don't have it.

1. [30 pts] Let $\Sigma = \{0, 1, \#\}$, and $L = \{x\#y \mid x, y \in \{0, 1\}^*$ and when interpreted as binary numerals, y is the square of $x\}$. E.g., $011\#1001$ is in L ($3^2 = 9$), but $1\#10$ is not. In this problem you will give *three* proofs that L is not regular.
   (a) Use a cut-and-paste style proof as on slides 10-12 of the lecture notes, involving many equal-length strings. Please note that I would give myself a grade of about 6/10 if I turned in those slides as a solution to a problem like this, with the note "right idea, but too terse; justify many steps more fully." (Hopefully, a transcript of my verbal justification would score higher, but the TA's, unlike your charming professor, are SO MEAN, that you'd best be careful. Ditto for all following problems ...)
   (b) Repeat the proof using the "increasingly long simple strings" style found on slides 13-15. Again, justify the various steps more carefully than I did on the slides...
   (c) Prove it again using the Pumping Lemma (Thm 1.70). Note that a proof based on the pumping lemma should make *no mention* of DFAs or states, unlike the methods in the previous two parts.

In subsequent problems, you may use any sound method, including any of the three above.

2. Let $\Sigma = \{a, b\}$.
   (a) Prove that $G = \{w \in \Sigma^* | w$ is a palindrome$\}$ is not regular.
   (b) Prove that $F = \{w \in \Sigma^* | w$ is not a palindrome$\}$ is not regular. [Hint: see exercise 1.14 .]
3. 1.30
4. Extra Credit: let $\Sigma = \{0, \ldots, 9\}$ and $L = \{y \in \Sigma^* \mid \exists x \in \mathbb{N}$ s.t. $x^2 = y$ when interpreted as a decimal numeral$\}$. Prove that L is not regular. This is the analog of Example 1.76 for perfect squares in decimal (but the proof is more complex/different). Although it doesn't matter, for definiteness assume the leftmost digit of the string is the high-order digit, as usual. You may also do it for binary numerals if you prefer, but I think decimal is slightly easier to think about.
5. Extra Credit: For any two strings $x, y \in \Sigma^*$ of equal length, define the *Hamming distance* between them $H(x, y)$ to be the number of indices i such that the i^{th} letters of x and y disagree. E.g., $H(x, y) = 0 \Leftrightarrow x = y$, and for any x there are $|x| * (|\Sigma| - 1)$ strings y such that $H(x, y) = 1$; i.e., y disagrees with x in any one of $|x|$ positions. Let $L = \{xy \mid H(x, y) = 1,$ where $|x| = |y|, x, y \in \Sigma^*\}, |\Sigma| \geq 2$. Show that L is not regular.

6. Extra Credit: 1.54

7. Extra Credit: referring to slide 32, fill in one or more of the remaining boxes (labeled "(exercise)") by sketching algorithms for those problems and analyzing their run times. I believe one of the three is linear time, and the other two are not. For the two "$x \in L$" problems, assume $|x| = n$ and let m be an appropriate measure of the size of the NFA or regexp, and estimate the run time as a function of both parameters, e.g. "$O(m + n)$" or "$O(m/n)$" or whatever.