CSE 322

Intro to Formal Models in CS

Homework #4

Due: Monday, Nov 8

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Again please place each problem in a separate, stapled, turn-in bundle, with your name on each.
Text problems below are on pages 83-93 of Sipser, *US second edition*; see online scanned versions if you don't have it.

1. 1.43. The last problem on the midterm was the special case where the dropped letter was always the 1st letter. Make sure you understand that first (although it is simpler than this problem). As a suggestion, you might want to think about the case where the letter being dropped is, say, the 2nd letter. Or perhaps the 10th letter.

2. 1.51. "Whenever" means "if and only if."

3. Suppose L is regular and M is a DFA recognizing L.

   (a) Following the definition in 1.51, show that if two strings x and y are distinguishable by L, then the state reached my M after reading x must be different from the state reached after reading y.

   (b) Prove: if there are k strings $\{x_1, \ldots, x_k\}$ every pair of which are distinguishable with respect to L, then M has at least k states. [Hint: pigeon hole principle.]

   (c) The point of the above is that it sometimes gives us a way to prove *impossibility* results — since the statements above apply to *any* DFA for L, they might allow us to prove that it is impossible for a DFA with a small number of states to recognize L. Use these ideas to prove that *any* DFA recognizing the language in Example 1.30 (3rd letter from the right end is 1) must have at least 8 states. Thus the machine in Fig. 1.32 is optimal. (Note that it is not sufficient to argue that various potential "optimizations" to fig 1.32 don't succeed in reducing its number of states, since in principle the best machine might look radically different, and "tweaking" 1.32 isn't guaranteed to find it.)

   (d) Extra Credit. Generalize part (c) to show that the "powerset construction" is nearly optimal, in the sense that for each $n \geq 3$, there is a language recognized by an $n + 1$ state NFA, but by no DFA with fewer than 2^n states.