CSE 322
Intro to Formal Models in CS
Homework #1
Due: Friday 8 Oct 10

W. L. Ruzzo 30 Sep 10

Read the section on "Collaboration" on the course home page.

On all assignments, you need to break your solution into *three* separate packets, so we can keep all three TAs out of mischief. *Staple* each packet and put your *name* on each. For this assignment:

- Problems 1–3 in packet 1
- Problem 4 in packet 2
- Problem 5 in packet 3.

Turn in your solutions on paper in class. If you can't attend, bring the papers to Ruzzo's office before class, or submit a copy electronically (e.g., a scanned version) using the Catalyst drop box linked from the course home page. FAX is OK (206-543-2969) if you label it clearly as destined for Ruzzo *and* send email to let him know to look. For electronic submissions, make sure the 3 packets outlined above start on separate sheets, with your name on those three sheets, at least.

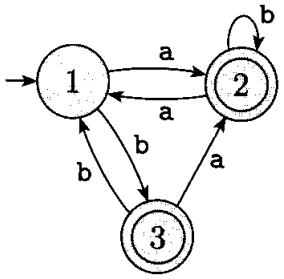
In all cases, homework is due by the begining of class on the due date.

1. If A is a set with n elements, how many elements are in the powerset of A? Explain your answer.

   Extra Credit: prove it by induction.

2. Consider the undirected graph $G = (V, E)$ where the set of nodes is $V = \{1, 2, 3, 4\}$ and the set of edges is $E = \{\{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 4\}, \{2, 4\}\}$. Draw the graph G. What is the degree of node 1? of node 3? Indicate a path from node 3 to node 4 in your drawing.

3. For the DFA, M, with the given state diagram:



   (a) What is the start state of M?
   (b) What is the set of accept states of M?
   (c) What sequence of states does M go through on input `abab`?
   (d) Does M accept the string `abab`? Explain why or why not.
   (e) Does M accept the string ε? Explain why or why not.

4. Give state diagrams for DFAs recognizing the following languages. The alphabet is $\Sigma = \{0, 1\}$ in all cases.

   (a) $\{w \mid w$ begins with a 1 and ends with a 0$\}$.
   (b) $\{w \mid w$ contains at least three 1s$\}$.
   (c) $\{w \mid \exists x, y \in \Sigma^*$ s.t. $w = x0101y$, i.e., w contains the substring 0101$\}$.
   (d) $\{w \mid$ the length of w is at most 5$\}$.

(e) $\{w \mid$ every odd position of w is $1\}$. (The first, third, ... letters of a string are odd positions; i.e., 1-based indexing, unlike the zero-based indexing common in programming languages.)

(f) The empty set.

(g) (Extra Credit) The complement of the language from part (c).

(h) (Extra Credit) The intersection of the languages from parts (b) and (e).

5. Using the following definition of string length,

$$|x| = \begin{cases} 0 & \text{if } x = \epsilon \\ |y| + 1 & \text{if } x = ya \text{ for some } y \in \Sigma^* \text{ and } a \in \Sigma. \end{cases}$$

prove, by induction on $|v|$, that

$$\forall\, u, v \in \Sigma^*,\ |uv| = |u| + |v|.$$