

CSE 322
Winter Quarter 2009
Assignment 6
Due Friday, February 13, 2009

All solutions should be neatly written or type set. All major steps in proofs must be justified. Please start each problem solution on a new page and put your name on every page.

1. (10 points) In this problem you will explore how to convert an ambiguous context-free grammar into an unambiguous one. A context-free grammar is ambiguous if there is some $w \in L(G)$ such that w has at least two distinct parse tree (or equivalently, w has at least two different leftmost derivations). Consider the context-free grammar $G = (\{S\}, \{a, b\}, R, S)$ where

$$R = \{S \rightarrow SS, S \rightarrow aSb, S \rightarrow ab\}.$$

- (a) Demonstrate that G is ambiguous by showing two distinct parse trees for some string in $L(G)$
(b) Give an alternative grammar for $L(G)$ that is not ambiguous.
2. (10 points) In this problem you will design a context-free grammar. Design a context-free grammars for the language

$$L = \{a^n b^m c^{2n+3m} : n, m \geq 0\}$$

and give a derivation of ab^2c^8 . For each nonterminal in your grammar explain what set of strings is derived by the nonterminal. That is, for each nonterminal A describe the language $\{w \in \{a, b, c\}^* : A \xRightarrow{*} w\}$.

3. (10 points) In this problem you will explore some closure properties of context-free languages.
- (a) Show that the context-free languages are closed under union. That is, given two context free grammars $G_1 = (V_1, \Sigma_1, R_1, S_1)$ and $G_2 = (V_2, \Sigma_2, R_2, S_2)$ construct a new grammar G such that $L(G) = L(G_1) \cup L(G_2)$.
- (b) Show that the context-free languages are not closed under intersection. You can use the fact that the language $\{0^n 1^n 2^n : n \geq 0\}$ is *not* context-free.
- (c) Show that the context-free languages are not closed under complement.