CSE 322
Winter Quarter 2009
Assignment 5
Due Friday, February 6, 2009

All solutions should be neatly written or type set. All major steps in proofs must be justified. Please start each problem solution on a new page and put your name on every page.

1. (10 points) In this problem you will apply the pumping lemma to show that a language is not regular. Consider the language $P = \{0^n : n \text{ is prime}\}$. So $P = \{0^2, 0^3, 0^5, 0^7, \ldots\}$. Use the pumping lemma to show that P is not regular.

2. (10 points) Consider the finite language $L_n = \{0,1\}^n$ which is the set of binary strings of lenth exactly n. Show that for all n, any NFA that recognizes L_n has at least $n+1$ states.

3. (10 points) Consider the language

$$L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$

   (a) Show that L is not regular.

   (b) Show that L actually satisfies the pumping lemma for regular languages. To do so you must find he p in the lemma then show that: if $s \in L$ and $|s| \geq p$ then your can partition s into the three parts satisfying the three parts of the lemma. There will be several cases.

4. (10 points) For this problem we define a new kind of expression called *star-free regular expressions* over an alphabet Σ. We define them recursively by:

   - ϕ and $a \in \Sigma$ are star-free over Σ.
   - If S and T are star-free over Σ then so are $S \cup T$, ST, and $\neg S$.

As usual the language described by ϕ is the empty set, a is $\{a\}$, $S \cup T$ is the union of the languages described by S and T, and ST is the concatenation of the languages described by S and T. Finally, the language described by $\neg S$ is the complement of the language described by S with respect to Σ^*. For example $\neg\phi$ describes Σ^*. The star-free regular languages are those described by star-free regular expressions.

(a) Show that $\{\varepsilon\}$ is star free.

(b) Show that every finite language over Σ is star-free.

(c) Show that the language described by $(01)^*$ cna be described by a star-free regular expression.

(d) (Extra Credit, 10 points) Show that $(00)^*$ cannot be described by a star-free regular expression. Thus star-free regular expressions are less powerful then normal regular expressions. If you work on this for an hour and get nowhere, that would very normal. If you are serious about solving this, please contact me first.