CSE 322
Winter Quarter 2009
Assignment 4
Due Friday, January 30, 2009

All solutions should be neatly written or type set. All major steps in proofs must be justified. Please start each problem solution on a new page and put your name on every page.

1. (10 points) In this problem you will study the relationships between prefixes, suffixes, and reversals. Recall that u is a ***prefix*** of x, if for some string y, $x = uy$. Similarly, v is a ***suffix*** of x, if for some string y, $x = yv$. Define the following operations on languages.

$$\text{Pre}(L) = \{u : u \text{ is a prefix of } x \text{ for some } x \in L\}$$
$$\text{Suff}(L) = \{v : v \text{ is a suffix of } x \text{ for some } x \in L\}$$

   (a) Given a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that accepts L, construct a DFA M' with the property that $L(M') = \text{Pre}(L)$.

   (b) Show that $\text{Suff}(L)$ is definable in terms the reversal of a language and the prefix of a language operations. (Hint: what I mean by definable is what I mean when I say that intersection is definable in terms of union and complement, using DeMorgans Law: $A \cap B = \overline{\overline{A} \cup \overline{B}}$.)

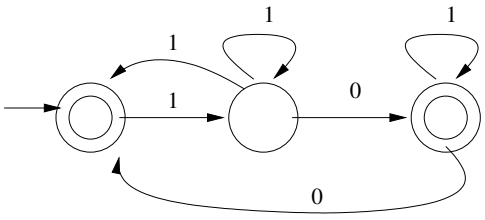
   (c) Explain, using (b) and other facts you know, why if L is accepted by a DFA, then $\text{Suff}(L)$ is accepted by a DFA.

2. (10 points) In this problem you can practice some of the constructions we are doing. Consider the regular expression $\alpha = (00 \cup 01)^*11$.

   (a) Carefully construct the equivalent NFA (a state diagram) that accepts the language defined by α. The construction is shown in the proofs of theorems 1.45, 1.47, and 1.49 in the book. Do not take any shortcuts.

(b) From the result in (a) above construct the equivalent NFA that has no ε-transitions. The main idea in the construction is to create a new transition on symbol a from state p to q if there is a sequence of ε-transitions from p to some state r, followed by a transition on symbol a from r to q. The set of final states may have to be increased. Remove all the unreachable states.

3. (10 points) In this problem you will practice the process of converting a finite automaton into an equivalent regular expression. Consider the following NFA.



Show each of the steps in the state elimination method for converting the NFA into a regular expression. For each intermediate GNFA, the regular expressions on each transition may be simplified to keep the regular expression as small as possible.

4. (extra credit, 10 points) Given a language L, define

$$\text{Half}(L) = \{x : \text{ for some } y, xy \in L \text{ and } |x| = |y|\}$$

Show that if L is regular then so is Half(L).