Reading Assignment: Sipser 2.2,2.3

1. Find a pushdown automaton which recognizes the language

$$\{a^m b^n | n \le m \le 2n, m, n \ge 0\}$$

You may give your answer as a state diagram (do not use the shorthand we used in class for pushing multiple symbols onto the stack in your transition function.) You need not turn in a proof of correctness, but you should right a description describing why your PDA recognizes the language.

2. (a) Convert the following CFG into a PDA

$$S \to (S) | [0S] | SS | \varepsilon$$

For your answer you may give a state diagram, but you expand out all of the states and not use the shorthand we used in class for pushing multiple symbols onto the stack.

- (b) Now, for the PDA you have constructed, show a sequence of configurations (state and stack) which would cause your PDA to accept ()[0[0()[0]]].
- 3. Give a CFG for the language recognized by the following pushdown automata:

$$\begin{array}{c} 1,\varepsilon \rightarrow 1 & 2,1 \rightarrow \varepsilon & 3,\varepsilon \rightarrow \varepsilon \\ \hline q_1 \xrightarrow{\varepsilon,\varepsilon \rightarrow \$} \boxed{q_2} \xrightarrow{\varepsilon,\varepsilon \rightarrow \varepsilon} \boxed{q_3} \xrightarrow{\varepsilon,\$ \rightarrow \varepsilon} \boxed{q_4} \\ \downarrow \varepsilon,\varepsilon \rightarrow \varepsilon \\ \hline q_5 \xrightarrow{\varepsilon,\varepsilon \rightarrow \varepsilon} \boxed{q_6} \xrightarrow{\varepsilon,\$ \rightarrow \varepsilon} \boxed{q_7} \\ \bigcirc & \bigcirc \\ 2,\varepsilon \rightarrow \varepsilon & 3,1 \rightarrow \varepsilon \end{array}$$

You may do this in any manner you like (i.e. you do not have to follow the method used in the book to convert PDAs to CFGs.)

- 4. For any language A, let $PREFIX(A) = \{x | xy \in A \text{ for some string } y\}$. Show that the class of context free languages is closed under the PREFIX operation. Assume you are working over a fixed alphabet Σ . (Note: the method used in class may not be the best way to prove this.)
- 5. Extra Credit For the glory, not for the points! Prove that any grammar for the language $A = \{a^i b^j c^k | i = j \text{ or } j = k\}$ must be ambiguous.