# CSE 322 Autumn 2009
# Assignment #4

Due: Friday, October 30, 2009 in class

**Reading assignment:** Finish reading section 1 of Sipser's book and read handouts on the Myhill-Nerode Theorem and Minimization of Finite Automata.

**Problems:**

1. Use the pumping lemma to prove that the following languages are not regular.
   (a) $\{ww^R \mid w \in \{0,1\}^*\}$.
   (b) $\{0^n 1^m 0^n \mid m, n \geq 0\}$.
   (c) $\{a^n \mid n \text{ is prime}\}$.
2. Sipser's book 2nd edition Problem 1.48 (1st edition Problem 1.41).
3. Use the method from the Myhill-Nerode handout to prove that the following languages are not regular.
   (a) $\{www \mid w \in \{a,b\}^*\}$.
   (b) $\{0^n 1^m 0^n \mid m, n \geq 0\}$.
   (c) $\{w \mid w \neq w^R, w \in \{0,1\}^*\}$.
4. Show that the language
   $$\{a^i b^j c^k \ : \ i, j, k \geq 0, \text{ and if } i = 1 \text{ then } j = k\}$$
   satisfies the conclusion of the pumping lemma (and therefore the pumping lemma cannot prove that it is not regular). Show that it is not regular using another method. Explain why this does not contradict the pumping lemma.
5. On the 2nd homework, problem 4 asked you to describe an NFA with $k+1$ states that recognizes the language $C_k = \Sigma^* a \Sigma^{k-1}$ where $\Sigma = \{a, b\}$. Prove that any *DFA* that recognizes $C_k$ must have at least $2^k$ states by showing that $\equiv_{C_k}$ has at least $2^k$ different equivalent classes; i.e., there are at least $2^k$ strings, no two of which are equivalent w.r.t. $\equiv_{C_k}$.
6. (Extra Credit) A set $S$ of non-negative integers is *semi-linear* if and only if $S = \{a + ib \mid i \geq 0\}$ for some integers $a, b \geq 0$. (For example the set of all positive integers congruent to 3 modulo 7 is semi-linear using $a = 3$ and $b = 7$ as is $\{34\}$ using $a = 34$ and $b = 0$.) Prove that $A \subseteq \{0\}^*$ is regular if and only if $A = \{0^n \mid n \in T\}$ where $T$ is the union of a finite number of semi-linear sets. (Hint: use the pumping lemma for regular languages or the Myhill-Nerode theorem.)