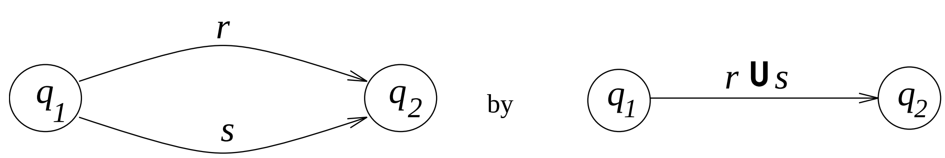
CSE 322
Introduction to Formal Models in Computer Science

Regular expressions from Finite Automata

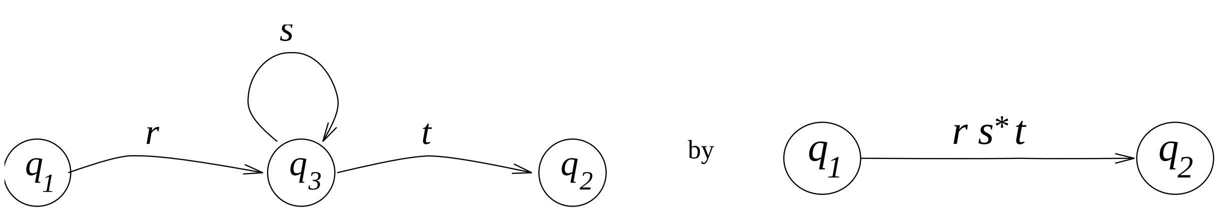
The key idea for the construction that creates a regular expression from a finite automaton is to allow edge labels that are regular expressions. Sipser calls this a Generalized Finite Automaton but his formal description is more constrained than I think is convenient. (The main new thing in the definition here is to allow parallel edges between states). The intuition is that in following an edge labelled by regular expression r, some prefix of the input remaining to be read is in $L(r)$, the language represented by r and following the edge means reading such a prefix. A string x will be accepted if and only if there is some path from the start state to a final state whose labels concatenated together form a regular expression whose associated language contains x. Notice that our standard NFA's and DFA's are special cases of this where all our regular expressions turn out be some $a \in \Sigma$ in the DFA case and either $a \in \Sigma$ or ε in the NFA case.

For the construction we first add a new start state and a new final state connected to (resp. from) the old ones via ε-moves. (This is so that no start or final state is on a cycle.) There are only two rules which we apply until the graph is reduced to a single labelled edge which will have the regular expression on it.

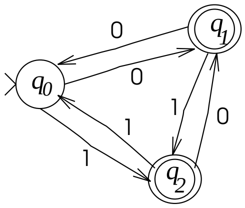
Rule 1. Combination of Parallel Edges: If q_1 and q_2 are any two states (possibly $q_1 = q_2$) then replace

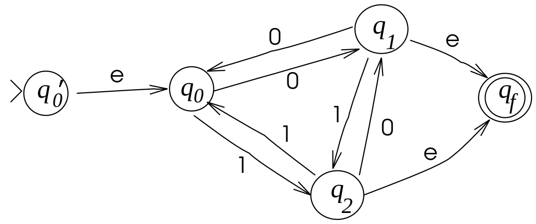


Rule 2. Removal of States: If q_3 is not either the new start state or the new final state then for **every** pair of states q_1 and q_2 (again possibly $q_1 = q_2$) replace

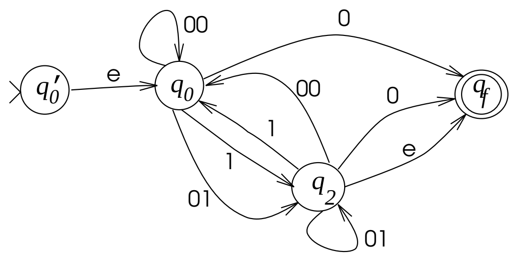


Turn page over for an example (e stands for ε in the example).

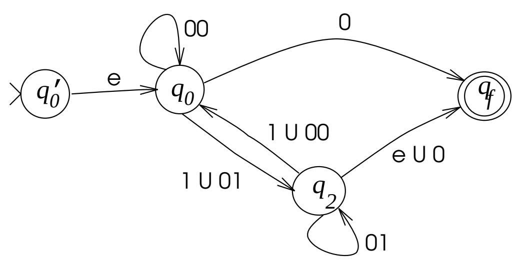




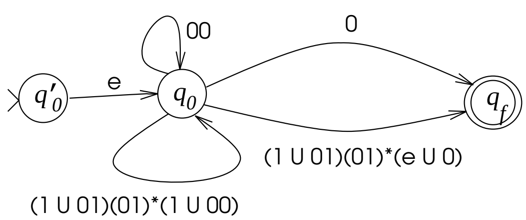
Rule 2 to q_1



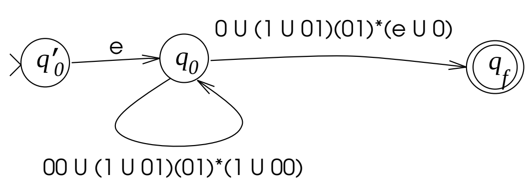
Rule 1 to edges



Rule 2 to q_2



Rule 1 to edges



Rule 2 to q_0