CSE 322: Formal Models in Computer Science — **Assignment #1**

March 31, 2008 — Due: **Friday**, April 11, 2008

Reading Assignment: 0.1-0.4 (review) and 1.1-1.2

Problems:

1. Sipser's book, Exercise 1.3 (same in both editions.) Make sure you include everything that a state diagram should include!

2. The rule for valid names for variables in C programs is the following. Variables must begin with a character (that is, a letter in the English alphabet) or underscore and may be followed by any combination of characters, underscores, or the digits $0 - 9$. Design a DFA that accepts strings that are valid variable names (For simplicity assume that $\Sigma = \{< c >, < d >, < u >, \#\}$ where $< c >$ denotes a character, $< d >$ denotes a digit, and $< u >$ denotes and underscore, and # denotes any other possible ASCII character.

3. Give state diagrams of DFAs recognizing the following languages. In each parts the alphabet is $\Sigma = \{0, 1\}$. As documentation for you DFA, for each state, give a brief informal description of the set of strings which reach this state.

   (a) $\{ w \mid w$ contains at least three 1s. $\}$

   (b) $\{ w \mid w$ has length at least 3 and its third symbol is a 0. $\}$

   (c) $\{ w \mid w$ has an even number of 0s and an odd number of 1s. $\}$

   (d) $\{ w \mid w$ begins with a 1, and which, interpreted as the binary representation of a positive integer, is divisible by 4 $\}$.
   For this problem assume that the DFA starts reading the string from its most signicant bit. For example if $w = 1000$, then w is the binary representation of the (decimal) number 8 (and thus, is in the language). and the DFA starts by reading the bit 1.

4. The reversal of a string w denoted by w^R, is the string when you look at it backwards: for example, $homer^R = remoh$. Here is the formal inductive definition (where the alphabet is Σ):

   Base case. If $w = \epsilon$, then $w^R = \epsilon$.
   Inductive step. If $w = va$ for $v \in \Sigma^*$ and $a \in \Sigma$, then $w^R = av^R$.

   Prove by induction (on the length of y) that for all strings $x, y \in \Sigma^*$,

   $$(xy)^R = y^R x^R$$

5. **Extra Credit** (minimal points, do it for the glory!) Sipser's book, Problem 1.37 in second edition (Problem 1.30 in the first edition.)