CSE 322 Winter 2006

<u>Homework Assignment # 5</u>

Due Date: Friday, Feb 24 (at the *beginning* of class)

1. (10 points: 5 points each) Consider the CFG G_4 in Exercise 2.1 in the textbook (both editions). Give <u>parse trees</u> and <u>leftmost derivations</u> for the following strings:
   a. $((a) \times a)$
   b. $a + (a \times (a + a))$

2. (35 points: 7 points each) Let $\Sigma = \{0,1\}$. Give CFGs that generate the following languages over Σ:
   a. {w | w contains the substring 10 and ends in 0}
   b. {w | w contains an odd number of 0's <u>and</u> at least two 1's}
   c. the set of all strings except the empty string and the string 0
   d. $\{1^i01^j01^k \mid i, j \geq 1 \text{ and } k = i + j\}$
   e. $\{ w \mid w = w_1w_2 \text{ where } w_1, w_2 \in \Sigma^*, |w_1| = |w_2| \text{ and } w_1 \neq w_2 \}$

3. (15 points: 5 points each) Show that context-free languages are closed under the following operations:
   a. concatenation
   b. string reversal
   c. Suffix, where for any language L, $\text{Suffix}(L) = \{y \mid y \in \Sigma^* \text{ and } xy \in L \text{ for some string } x \in \Sigma^*\}$

4. (20 points: 10 points each) Let $\Sigma = \{0,1\}$.
   a. Show that the following CFG is ambiguous:
      $S \rightarrow ABA \qquad A \rightarrow 0A \mid \varepsilon \qquad B \rightarrow 1B \mid \varepsilon$
   b. Give an <u>equivalent unambiguous</u> CFG.

5. (20 points: 10 points each) Give <u>informal descriptions</u> (as in Example 2.16 in the textbook (2.10 in the 1st ed.)) and <u>state diagrams</u> of pushdown automata (PDA) for the following languages over $\Sigma = \{0,1\}$:
   a. $\{w \mid$ the number of 0s in w is two times the number of 1s in $w\}$
   b. $\{0^i10^j10^k \mid i = j \text{ or } j = k\}$