CSE 322 Winter 2006

Homework Assignment # 2

Due Date: Friday, January 20 (at the *beginning* of class)

1. (20 points) Prove that for any two integers x and y, their product xy is even if and only if at least one of the integers x or y is even.

2. (20 points) There are 57 students enrolled in CSE 322. An unauthorized survey by Nyke™ has revealed that the students own a total of 289 pairs of shoes. Show that at least one student owns more than 5 pairs of shoes (you know who you are!). Hint: Use the pigeonhole principle.

3. (20 points) Let A and B be any two infinite subsets of Σ^* where $\Sigma = \{0,1\}$. Show that $A \cup B$ is countably infinite. Hint: Use dovetailing.

4. (20 points) Let **F** be the set of all functions $f: \Sigma^* \rightarrow \{0,1\}$ where $\Sigma = \{a, b, \ldots, z\}$. One example of such a function is the function that maps all "legitimate" English words (according to your favorite dictionary) to 1 and all other words to 0. Show that **F** is uncountable.

5. (20 points) A bug has been found in software for warrantless wiretapping at <name-withheld> agency. The software engineers at the agency suspect the bug is in the following code fragment but they haven't been able to prove it. Knowing the reputation of UW CSE undergrads in computer science theory, they've approached you (discreetly, of course) to substantiate their claim. Prove that the code fragment is in fact correct i.e. that for any natural number `n`, the code prints out n^2 (hint: consider the value of `s` and use induction on the number of times the `while` loop is executed):

```
s = 0;
i = 0;
while (i < n) {
 s = s + n;
 i++;
}
print(s);
```