Name: _____
Student ID: _____

CSE 322 Autumn 2001: Sample Final Exam

(some parts based on L. Ruzzo's sample final, Autumn 2000)
(closed book, closed notes except for 1-page summary)
Total: 150 points, 6 questions. Time: 1 hour and 50 minutes

Instructions:

1. Write your name and student ID on each sheet. Write or mark your answers in the space provided. If you need more space or scratch paper, you can get additional sheets from the instructor. Make sure you write down the question number and your name/id on any additional sheets.
2. Read all questions carefully before answering them. Feel free to come to the front to ask for clarifications.
3. *Hint 1*: You may answer the questions in any order, so if you find that you're having trouble with one of them, move on to another one that seems easier.
4. *Hint 2*: If you don't know the answer to a question, don't omit it - do the best you can! You may still get partial credit for whatever you wrote down. Good luck!

1. (25 points)
Let L be the set of strings in {a,b}* such that each a, if any, has two b's immediately to its right. Give:

(a) a finite automaton accepting L.

(b) a regular expression denoting L.

(c) a context-free grammar generating L.

You do *not* need to follow the constructions given in the book for converting one of these forms to the other; you may just give a direct answer for each part.

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2. (25 points)
Define a well-formed list to be either the single "atom" a, or a sequence of one or more well-formed lists separated by commas and enclosed in parentheses. For example, "a" is a list, as are "(a)" and "(a,a,((a)))" but "((),aa)" is not (for two reasons).

(a) Give a context-free grammar that generates all such lists.

(b) Give a parse tree for the list "(a,a,((a)))" with respect to your grammar.

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3. (25 points)

Let G = (V,Σ,R,S) be a context-free grammar, where Σ = {w, c, {, }, ;, a}, V = Σ ∪ {S} and R is given by:

S → wcS
S → {S}
S → S;S
S → a

(This grammar models some statements in C-like languages, where w represents **whil**e, c represents a Boolean condition, {…} represents a compound statement, and a represents a non-**while** statement.)

(a) Show two distinct leftmost derivations, and the corresponding parse trees, for the string: wca; a

(b) Is *G* ambiguous? Why or why not?

(c) Convert the CFG G above to a PDA accepting the same language.

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(d) Show an accepting computation of your PDA for the input: wc{a}. Be sure to show the full configuration at each step — state, portion of input string that remains to be read, and contents of stack. If your PDA is nondeterministic, there may be more than one possible accepting computation. Show only *one* of them. [*Clearly indicate the top of the stack*.]

	State	Input to be read	Stack
1.			
2.			
3.			
4.			
:			
:			

4. (25 points)

a. Show that the language L = {ww| $w \in$ {0,1}*} is not a context-free language.

b. Show that L is decidable. (Give an implementation-level description of a Turing machine that decides L).

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5. (25 points)

a. Show that the class of context-free languages is *not closed* under intersection.

b. Show that the class of decidable languages is closed under complementation.

c. Show that the class of Turing-recognizable languages is closed under concatenation.

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6. (25 points)

Prove that the language $EMPTY_{TM}$ = {<M> | M is a TM and L(M) = ∅} is undecidable. Use a reduction from A_{TM} = {<M,w> | M is a TM that accepts w} to $EMPTY_{TM}$.