Using the pumping lemma

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Announcements

- Turn in H/W # 3
- Handouts
  - H/W #4
  - Handout on Myhill-Nerode theorem
    - Next lecture
  - Solns to # 2, if you did not pick up one last time
- Graded H/W # 2 at the end of the lecture
- No puzzle today

Statement of the pumping lemma

- If L is regular then
  - $\exists$ integer $p \geq 1$
  - $\forall$ strings $s \in L$ with $|s| \geq p$
  - $\exists$ strings x,y,z satisfying s=xyz with
    - $|xy| \leq p$
    - $|y| > 0$
  - $\forall$ integer $i \geq 0$, $xy^iz \in L$

The "proof"

- L is regular
- L is accepted by a DFA with (say) p states
- Consider any string s in L with $|s| \geq p$
  - The "walk" in the DFA must contain a cycle
- Repeating the cycle arbitrary number of times will give new strings that are also in L

Contrapositive of implication

- $A \Rightarrow B$ is same as $\neg B \Rightarrow \neg A$
- Recall:
- $\neg(\forall x\ A(x))$ is same as $\exists x\ (\neg A(x))$
- $\neg(\exists x\ B(x))$ is same as $\forall x\ (\neg B(x))$

Contrapositive of the pumping lemma

- If L is regular then
  - $\exists$ integer $p \geq 1$
  - $\forall$ strings $s \in L$ with $|s| \geq p$
  - $\exists$ strings x,y,z satisfying s=xyz with
    - $|xy| \leq p$
    - $|y| > 0$
  - $\forall$ integer $i \geq 0$, xy^iz in L

- If
  - $\forall$ integer $p \geq 1$
  - $\exists$ string $s \in L$ with $|s| \geq p$
  - $\forall$ strings x,y,z satisfying s=xyz with
    - $|xy| \leq p$
    - $|y| > 0$
  - $\exists$ integer $i \geq 0$, xy^iz not in L
- then L is not regular

Applying the pumping lemma

- Is a you will game with the devil

- Your job is prove L is not regular and devil will try to foil your attempt

Using the pumping lemma

- If
  - $\forall$ integer $p \geq 1$
  - $\exists$ string $s \in L$ with $|s| \geq p$
  - $\forall$ strings x,y,z satisfying $s=xyz$ with
    - $|xy| \leq p$
    - $|y| > 0$
  - $\exists$ integer $i \geq 0$, xy^iz not in L
- then L is not regular

- Round 1:
  - Devil picks p

- Round 2: You
  - You pick $s \in L$
- Round 3:
  - Devil picks x,y,z

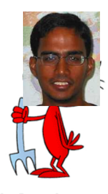
- Round 4:
  - You pick i

You win if xy^iz not $\in L$

You want to win **no matter** what the devil does

In case you think the devil is cute…

- Assume you are giving your exam
- The devil looks somewhat like this



Seriously, what is going on ?

- We want to prove L is not regular
  - Recall p is the number of states of a DFA
  - We don't know the DFA
    - Cannot assume anything about p
  - If we do not know the DFA
  - We do not know where it would cycle
    - Cannot assume anything about x, y, z (other than $|xy| \leq p$ and $|y|>0$)

Using the pumping lemma

- If
  - $\forall$ integer $p \geq 1$
  - $\exists$ string $s \in L$ with $|s| \geq p$
  - $\forall$ strings x,y,z satisfying $s=xyz$ with
    - $|xy| \leq p$
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You want to win **no matter** what the devil does

Let's prove some stuff now…

- $L = \{ 0^n1^n \mid n \geq 0 \}$ is not regular