CSE322: Formal Models in Computer Science Spring 2006

PROBLEM SET 7

Due Friday, May 26, 2006, in class

Reading Assignment: Read Sections 3.1, 3.3 and 4.1 of Sipser's text.

Instructions: The basic instructions are the same as in Problem Set 1.

There are **SIX** questions in this assignment. Again, it never hurts to start early. Also **do not forget** to mention the names of your collaborators in your homework.

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1. (**Bonus**) (5 points) Show that the following context-free language is inherently ambiguous: $L_1 = \{0^i 1^j 2^k \mid i = j \text{ or } j = k, \text{ where } i, j, k \geq 0\}$.

2. (10 points) Let G be a context-free grammar in Chomsky normal form that contains k variables. Show that if $L(G)$ contains a string of length greater than 2^k, then $L(G)$ is infinite. (that is, the set of strings in $L(G)$ is infinite.)

   (*Hint*: It might help to think of the parse trees like we did in the proof of the pumping lemma.)

3. (*) (10 points) Using the method covered in class, convert the PDA in Figure 2.19, Pg. 114 in Sipser's text (Fig 2.8, Pg. 106 in First edition) to a CFG. Show your steps.

4. ($2 \times 10 = 20$ points) Use the pumping lemma to prove that the following languages are not context-free.

   (a) $L_2 = \{0^n 1^n 2^n 3^n \mid n \geq 0\}$. (The alphabet is $\{0, 1, 2, 3\}$.)

   (b) $L_3 = \{x_1 \# x_2 \# \cdots \# x_k \mid k \geq 2, \text{ each } x_i \in \{a, b\}^*, \text{ and } x_i = x_j \text{ for some } i \neq j\}$.
   (*Note*: L_3 is different from the language in the previous homework– the previous homework question needed $x_i = x_j^R$.)
   (*Hint*: Try and think of an s that has very few #.)

5. (10 points) Let L_4 be the language of all palindromes over $\{0, 1\}$ containing an equal number of 0s and 1s. Prove that L_4 is not context-free.

6. (**Bonus**) (10 points) Prove the following strengthening of the pumping lemma for CFLs, where we require that **both** the string v and y (where s is broken up as $s = uvxyz$) are non-empty.

   If L is a CFL, then there exists an integer $p \geq 1$ such that for all $s \in L$ with $|s| \geq p$, there exists a way to break it down as $s = uvxyz$, which satisfy the following properties:

   (a) For all $i \geq 0$, $uv^i xy^i z \in L$,

   (b) $v \neq \epsilon$ **and** $y \neq \epsilon$,

   (c) $|vxy| \leq p$.