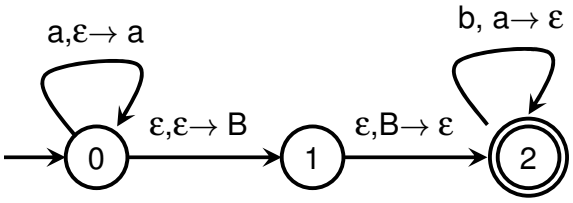
CSE 322
Introduction to Formal Models in Computer Science
Example of PDA to CFG conversion

Lemma 2.27 (Lemma 2.15 in 1st edition) of Sipser's text describes a general conversion from PDA's to CFG's. We will apply it to the following PDA which accepts the language $\{a^n b^n \mid n \geq 0\}$ and has all the properties required for the direct construction.



For the general construction there would be 9 variables A_{ij} for $i, j \in \{0, 1, 2\}$. However we only need to include those A_{ij} such that there is a path from i to j in the PDA diagram. This means that we only need to include rules involving $A_{00}, A_{01}, A_{02}, A_{11}, A_{12}, A_{22}$.

The start symbol is A_{02}. The rules involving only these symbols are:

$$
\begin{aligned}
A_{00} &\to A_{00}A_{00} \mid \varepsilon \\
A_{01} &\to A_{00}A_{01} \mid A_{01}A_{11} \\
A_{02} &\to A_{00}A_{02} \mid A_{01}A_{12} \mid A_{02}A_{22} \\
A_{11} &\to A_{11}A_{11} \mid \varepsilon \\
A_{12} &\to A_{11}A_{12} \mid A_{12}A_{22} \\
A_{22} &\to A_{22}A_{22} \mid \varepsilon \\
A_{02} &\to aA_{02}b \\
A_{02} &\to A_{11}
\end{aligned}
$$

where the next to last rule comes from pairing up the two self loop arcs and the last rule comes from pairing up the other two arcs.