CSE 322: Formal Models in Computer Science Winter 2005

PROBLEM SET 8

Due Friday, March 11, 2005, in class

Reading assignment: Sections 3.1, 3.3, 4.1, 4.2 of Sipser's text.

There are **FOUR** questions. Each question is worth **15 points**.

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1. (a) Show that decidable languages are closed under union, intersection, and complementation.

   (b) Show that Turing-recognizable languages are closed under union and intersection.

2. Show using a proof by diagonalization that the set of all infinite sequences over $\{0, 1\}$ is uncountable.

3. Let C be a language. Prove that C is Turing-recognizable if and only if there exists a decidable language D such that $C = \{x : \exists y(\langle x, y\rangle \in D)\}$.)
   Hint: For the only if part, it might help to think of y as the witness or proof that a string x is accepted by a Turing Machine. So think of what could serve as such a witness.

4. Define the language

$$A = \{\langle M\rangle \mid M \text{ is a DFA that only accepts strings over } \{0, 1\} \text{ with an odd number of 1's}\} .$$

   Show that A is decidable.
   Suggestion: Theorem 4.4 of Sipser's book might be useful.