CSE 322: Formal Models in Computer Science Winter 2005

PROBLEM SET 4

Due Friday, February 4, 2005, in class

Reading assignment: Handouts on Myhill-Nerode theorem and DFA minimization.

There are **SIX** questions, including an extra credit problem. Each question is worth **12 points**, except the extra credit problem which is worth 10 points.

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1. Are minimal state NFAs unique for every regular language? Justify your answer.

2. Consider the DFA $M = (\{a, b, c, d, e, f, g, h, i\}, \{0, 1\}, \delta, a, \{c, f, i\})$ where the transition function δ is given by the table:

	0	1
a	b	e
b	c	f
c	d	h
d	e	h
e	f	i
f	g	b
g	h	b
h	i	c
i	a	e

   Find the minimum-state DFA that is equivalent to M.

3. For an integer $k \geq 1$, define the language L_k over alphabet $\Sigma = \{0, 1\}$ as follows:

$$L_k = \{x \mid \text{ the } k\text{'th symbol from the right in } x \text{ equals } 1\} .$$

   (a) Design an NFA with $k + 1$ states that recognizes L_k.

   (b) Prove that no NFA with fewer than $k + 1$ states can recognize L_k.

   (c) Prove that every DFA that recognizes L_k must have at least 2^k states. (This shows that the exponential blow-up in the subset construction to convert NFAs to DFAs is sometimes inherent and unavoidable.)

4. Let Σ be an arbitrary alphabet and a any string in Σ^*. Define the language $\text{Suf}_a = \{xa \mid x \in \Sigma^*\}$, i.e., Suf_a consists of those strings which end with a. Suf_a is easily seen to be regular by a simple NFA construction with $|a| + 1$ states. How many states does the minimal DFA recognizing Suf_a have? Justify your answer.

5. Prove the following stronger version of the pumping lemma that gives a necessary and sufficient condition for a language to be regular: A language $A \subseteq \Sigma^*$ is regular **if and only if** there exists $p \geq 0$ such that for all $s \in \Sigma^*$ with $|s| \geq p$, there exist $u, v, w \in \Sigma^*$ such that $s = uvw$, $v \neq \epsilon$, and for all $z \in \Sigma^*$ and $i \geq 0$,

$$sz \in A \Longleftrightarrow uv^i wz \in A .$$

   Hint: Use the Myhill-Nerode theorem for the "if" part.

6. * (Extra Credit)

(a) A subset T of $\mathbb{N} = \{0, 1, 2, \ldots\}$ is said to be eventually periodic if there exist integers $m \geq 0$ and $p > 0$ such that for all $n \geq m$, $n \in T$ if and only if $n + p \in T$.
Let $L \subseteq \{0\}^*$ be a unary language. Prove that L is regular if and only if the set $\{n \mid 0^n \in L\}$, the set of lengths of strings in L, is eventually periodic.

(b) Let $L \subseteq \{0\}^*$ be an arbitrary unary language (in particular, L need not be regular). Prove that L^* is regular.
Hint: Use part (a) above by showing that the length of the shortest non-empty string in L can serve as a period (for the set of lengths of strings in L^*).