CSE 322: Formal Models in Computer Science Winter 2005

Problem Set 3

Due Friday, January 28, 2005, in class

Reading assignment: Sipser's book, sections 1.3 and 1.4.

Each question is worth **12 points**, except the Extra Credit problem which is worth 10 points.

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1. Consider the DFA $M = (\{q_0, q_1, q_2\}, \{0,1\}, \delta, q_0, \{q_0\})$, with the transition function: for $i = 0, 1, 2$, $\delta(q_i, 0) = q_{(2i \bmod 3)}$ and $\delta(q_i, 1) = q_{((2i+1) \bmod 3)}$.
   (a) Let A be the language that M recognizes. Can you give a simple description of A?
   (b) Use the finite automaton to regular expression conversion procedure we discussed in class to obtain a regular expression describing the language A.

2. For each pair of regular expressions below, prove that they describe the same regular language:
   (a) $(0^*1)^*0^*$ and $(0 \cup 1)^*$
   (b) $(01 \cup 0)^*0$ and $0(10 \cup 0)^*$

3. Consider a new kind of finite automaton called an *all-paths-NFA*. An all-paths-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ just like an NFA. The only difference is in the acceptance criterion: an all-paths-NFA accepts a string $x \in \Sigma^*$ if *every* possible computation of M on x ends in a state from F that accepts $x \in \Sigma^*$. (Note, in contrast, that an ordinary NFA accepts a string if *some* computation ends in an accept state.)
   (a) Argue that L is a regular language if and only if L is recognized by an all-paths-NFA.
   (b) Use part (a) to show that the set of regular languages is closed under intersection. That is, prove that if A, B are regular languages, then so is $A \cap B$.
   (c) Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA that recognizes language A. Let $N_{\text{flip}} = (Q, \Sigma, \delta, q_0, F')$ be the all-paths-NFA defined by taking $F' = Q - F$, and let A_{flip} be the language recognized by N_{flip}. How are A and A_{flip} related? Briefly justify your answer.

4. Show that the following languages are not regular. Please structure and write your arguments as clearly as possible.
   (a) $L_1 = \{0^i 1^j \mid i, j \geq \text{ and } i \neq j\}$
   (b) $L_2 = \{w \in \{0,1\}^* \mid w \text{ is a palindrome}\}$. (A palindrome is a string which reads the same forward and backward.)

5. Define the language

$$A = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}\ .$$

   (a) Show that A satisfies the three conditions of the pumping lemma, namely show that there exists $p \geq 1$ such that every $w \in A$, $|w| \geq p$, can be rewritten as $w = xyz$, $|xy| \leq p$, $y \neq \epsilon$, such that $xy^iz \in A$ for every $i \geq 0$.

(b) Is L regular? Why or why not? If not, why doesn't this contradict the pumping lemma?

6. * (Extra Credit) Let r and s be regular expressions where the language represented by r does not contain the empty string ϵ. Consider the equation $X = r \circ X \cup s$ (where $\circ$ stands for concatenation of regular expressions, and $\cup$ for union) with unknown variable X. Find a solution (namely, a regular expression) for X that satisfies the above equation and prove that this solution is *unique*.