CSE 322: Formal Methods in Computer Science
Winter 2005
Sample Midterm

1. (15 points) For each of the following statements, answer whether they are True or False by circling the appropriate choice. You do *not* need to justify your answer.

   (a) If $A \neq B \neq C$ are languages such that $A \cap B = C$ and B, C are both regular, then A must also be regular.

   (b) If L is regular, then so is the language $\{xy \mid x \in L,\ y \notin L\}$.

   (c) If L is regular, the minimum state DFAs for both L and $\bar{L}$ have the same number of states.

   (d) $b^*a^* \cap a^*b^* = a^* \cup b^*$

   (e) The minimum state DFA for the language $\{w \in \{a,b\}^* \mid w \text{ contains } abaab \text{ as a substring}\}$ has more than 6 states.

2. (30 points) Define the language $A = \{w \in \{0,1\}^* \mid$ the number of 0's minus the number of 1's in w is divisible by $3\}$.

   (a) Construct a DFA with only three states that recognizes A.

   (b) Prove that your DFA from Part (a) is optimal, i.e. three states are the minimum needed to recognize A.

   (c) Using the state elimination procedure described in class or otherwise, write down a regular expression that generates the language A.

3. (20 points) Using the pumping lemma for regular languages, prove that the language

$$\{a^n b a^m b a^{m+n} \mid n, m \geq 1\}$$

   is not regular.

4. (15 points) Prove or disprove: If $B \subseteq \{0,1\}^*$ is a regular language, then the language $C = \{x \in B \mid x \text{ does not contain } 1101 \text{ as a substring}\}$ is also regular.

5. (20 points) Design a context-free grammar for the language $\{0^i 1^j \mid j > i \geq 1\}$. Draw a parse tree for your grammar that derives the string $0^3 1^4$. Is this parse tree uniquely determined for your grammar?