CSE 322: Formal Models in Computer Science
Sample Final Exam: Winter 2005

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DIRECTIONS:

- Closed Book, Closed Notes
- Time Limit: 1 hour 50 minutes
- Attempt all questions
- Maximum possible score = 200 points

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1. (40 points) Answer True or False to the following questions and briefly JUSTIFY each answer.

   (a) If a PDA M actually pushes a symbol on its stack then $L(M)$ is not regular.

   (b) For every infinite regular language L over Σ there are strings $x, y, z \in \Sigma^*$ such that $|y| \geq 1$ and for every $k \geq 0$, $xy^kz \in L$.

   (c) There is no algorithm to tell, given an arbitrary string P whether or not P is the ASCII for a syntactically correct C program.

   (d) If L is a CFL and $L = K \cap R$ for a regular language R then K is a CFL.

   (e) If L is not a CFL and $L = K \cap R$ for a regular language R then K is not a CFL.

   (f) There is no algorithm to tell, given an arbitrary program P and an input x, whether or not P runs forever on input x.

   (g) If the stack in a PDA M only has capacity for 100 characters on any input then $L(M)$ is regular.

   (h) If L is accepted by some PDA M then L^R is accepted by some PDA M'.

   (i) There is no algorithm to tell, given an arbitrary CFG G and input x whether or not $x \in L(G)$.

   (j) There is an algorithm to tell, given an arbitrary NFA N, whether or not $L(N) = \emptyset$.

2. (30 points) Classify each of the following sets as:

   A Both Regular and Context-Free,

   B Context-Free, but not regular,

   C Neither context-free nor regular but membership in the language can be decided by an algorithm.

   D Undecidable

   You do not need to justify your answers.

(a) $\{a^nb^m \mid m = 2n+1\}$. .... A B C D

(b) $\{a^na^m \mid m = 2n+1\}$. .... A B C D

(c) $\{a^nb^m \mid m \equiv n \pmod 3\}$. .... A B C D

(d) $\{xcx^R \in \{a,b\}^* \mid x$ has an even number of a's$\}$. .... A B C D

(e) $\{a^ib^jc^k \mid k = i+j\}$. .... A B C D

(f) $\{a^ib^ja^k \mid k \neq i$ or $k \neq j\}$. .... A B C D

(g) $\{a^ib^ja^k \mid k \neq i$ and $k \neq j\}$. .... A B C D

(h) $\{a^nb^na^mb^m \mid m,n \geq 0\}$. .... A B C D

(i) $\{a^nb^ma^nb^m \mid m,n \geq 0\}$. .... A B C D

(j) $\{xy \in \{a,b\}^* \mid x \neq y\}$. .... A B C D

3. (25 points)Let $L = \{0^n1^m \mid m$ is an integer multiple of $n\}$.
Use the Myhill-Nerode theorem to show that L is not regular.

4. (30 points) Let $L = \{a^mb^nc^p \mid 0 \leq m < n < p\}$. Prove that L is not a context-free language.

5. (20 points) Let $G = (V, \Sigma, R, S)$ be the context-free grammar with the following set of rules:

$$S \rightarrow aSaSb \mid aSbSa \mid bSaSa \mid SS \mid e$$

   (a) Use one of the general constructions that convert a CFG to a PDA to give a PDA M such that $L(M) = L(G)$.

   (b) Show each step of a computation of your PDA (list the stack contents, state, and amount of input remaining) that accepts input *aabbaaaab*.

6. (25 points) Consider the grammar $S \rightarrow aS \mid aSbS \mid \epsilon$.

   (a) This grammar is ambiguous. Show in particular two parse trees and two leftmost derivations for the string *aab*.

   (b) What language does the grammar generate? (No justification necessary.)

   (c) Find an unambiguous grammar that generates the same language. (You don't have to prove unambiguity, but a one-sentence description of your main idea will help us understand your solution better.)

7. (30 points) For any context-free grammar $G = (V, \Sigma, R, S)$, we say that a nonterminal $A \in V$ is *useful* if there is a derivation $S \Rightarrow^* xAy \Rightarrow^* w$ for $w \in \Sigma^*$ and $x, y \in V^*$; otherwise it is *useless*. Suppose that G has the following rules:

$$S \rightarrow AC \mid BS \mid B$$

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\begin{array}{rcl}
A & \rightarrow & aA \mid aT \\
B & \rightarrow & CF \mid b \\
C & \rightarrow & cC \mid D \\
D & \rightarrow & aD \mid BD \mid C \\
E & \rightarrow & aA \mid BSA \\
T & \rightarrow & bB \mid b \\
U & \rightarrow & bA \mid a \mid Wb \\
W & \rightarrow & Ub \mid a \mid Bb
\end{array}
$$

(a) Which non-terminals of G are useful?

(b) Modify G to get an equivalent grammar G' whose nonterminals consist of precisely the nonterminals of G that are useful. (Don't remove any other non-terminals.)

(c) Describe a reasonably efficient algorithm that will remove all useless non-terminals from a grammar. Briefly argue (not formal proof necessary) why your algorithm works and thus show that any grammar is equivalent to one with only useful non-terminals.