CSE 322 Winter 2004
Assignment #8

Due: Friday, March 12, 2004

Reading assignment: Skim the start of Chapter 3 and pages 145,146. Read Chapter 4 of Sipser's text.

Problems:

1. Sipser's text, page 169, Exercise 4.7. (Note that infinite binary sequences are not strings since any string has finite length.)

2. Sipser's text, page 169, Exercise 4.8.

3. Sipser's text, page 169, Problem 4.11.

4. Define a *queue* automaton $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ where Q is the finite set of *states*, Σ is the *input alphabet*, Γ is the *queue alphabet*, q_0 is the *start state*, q_{accept} and q_{reject} are *accept* and *reject* states respectively, and

$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \to Q \times (\Gamma \cup \varepsilon).$$

A *configuration* of a queue automaton is an element of $Q \times \Sigma^* \times \Gamma^*$; configuration (q, y, z) represents that the current state is q, the remaining input is y, the current contents of the queue is z (with the left-most character on the left end of z).
If $\delta(p, a, A) = (q, B)$ where $A, B \in \Gamma \cup \{\varepsilon\}$ then its action on configurations is to take (p, ay, Az) to (q, y, zB).
Let $\Gamma = \Gamma' \cup \{\downarrow, \#\}$ where $\#, \downarrow \notin \Gamma', \Sigma$.

The following parts are the pieces of a design that would simulate a Turing machine by a queue automaton as outlined in class. Intuitively we think of the $\downarrow$ marking that the next character is where the head is and the $\#$ as marking one cell beyond the furthest visited spot on the tape after the end of the input.

(a) Design a sequence of transitions in δ that will take configuration (q_0, x, ε) to $(s, \varepsilon, \downarrow x\#)$ for $x \in \Sigma^*$.

(b) Design a sequence of transitions in δ that for fixed $p, q \in Q$, $a, b \in \Gamma'$ and any $z, z' \in (\Gamma')^*$ will take a configuration $(p, \varepsilon, z \downarrow az'\#)$ to configuration $(q, \varepsilon, zb \downarrow z'\#)$ if $z' \neq \varepsilon$ and to $(q, \varepsilon, zb \downarrow B\#)$ where $B \in \Gamma' - \Sigma$ is a special blank symbol if $z' = \varepsilon$.
(This corresponds to simulating a Turing machine move $\delta'(p, a) = (q, b, R)$.)

(c) Design a sequence of transitions in δ that for fixed $a, b \in \Gamma'$, $p, q \in Q$, and any $z, z' \in (\Gamma')^*, c \in \Gamma'$ will take configuration $(p, \varepsilon, zc \downarrow az'\#)$ to configuration $(q, \varepsilon, z \downarrow cbz'\#)$.
(This almost corresponds to simulating a Turing machine move $\delta'(p, a) = (q, b, L)$. The only exception is in part (d).)

(d) (Bonus) In a Turing machine if the read head tries to move left from the left end of the tape it just stays where it is. Generalize the previous construction from part (c) to simulate this behavior as well so that it also takes configuration $(p, \varepsilon, \downarrow\ az'\#)$ to configuration $(q, \varepsilon, \downarrow bz'\#)$.