CSE 322: The Last-but-one Homework Assignment

Due Date: Friday, May 28 (at the *beginning* of class)

1. (10 points) Convert the CFG G_4 in Exercise 2.1 (page 119) in the textbook to an equivalent PDA using the construction in the proof of Lemma 2.13. Give the state diagram showing all the states of your PDA.

2. (10 points) Convert the PDA M_2 in Example 2.10 (page 105) in the textbook to an equivalent CFG G_2 using the construction in the proof of Lemma 2.15.

3. (15 points) What is wrong with the following proof for showing that CFLs are closed under complement? (Hint: Give a counterexample to falsify the proof).
   Theorem: CFLs are closed under complement.
   Proof: Let L be any CFL and let L = L(M) for a PDA M = $(Q, \Sigma, \Gamma, \delta, q_0, F)$.
   Then, the complement of L is accepted by the PDA M' = $(Q, \Sigma, \Gamma, \delta, q_0, Q\text{-}F)$.
   Therefore, the complement of L is also a CFL.

4. (45 points) Use the pumping lemma to show that the following languages are not context free:
   a. $\{0^n 1^n 0^k \mid k \leq n\}$
   b. $\{w\#x \mid w, x \in \{0,1\}^* \text{ and } w \text{ is a substring of } x\}$
   c. $\{0^m \mid m \text{ is a perfect square i.e. } m = n^2 \text{ for some natural number } n \in N\}$

5. (20 points) Give the sequence of configurations that each of the following Turing machines enters when started on the indicated input strings:
   a. Turing machine M_2 from Example 3.4 in the textbook on: (i) input string 00 and (ii) input string 000
   b. Turing machine M_1 from Example 3.5 in the textbook on (i) input string 0# and (ii) input string 10#10