CSE 322 Spring 2004

<u>Homework Assignment # 2</u>

Due Date: Friday, April 16 (at the *beginning* of class)

1. (14 points) Give examples of each of the following if possible. If not possible, explain why.
   a. An uncountably infinite set A and a countably infinite set B such that B is a subset of A
   b. An uncountably infinite set A and an uncountably infinite set B such that their difference (A-B) is countably infinite
   c. Two different countably infinite sets whose intersection is countably infinite
   d. Two different uncountably infinite sets whose intersection is finite
   e. Two different uncountably infinite sets whose intersection is uncountably infinite
   f. Two countably infinite sets whose union is uncountably infinite
   g. An uncountably infinite set whose complement is countably infinite

2. (15 points) Show, using diagonalization, that the number of all possible functions f: $\{0,1\}^* \rightarrow \{0,1\}^*$ (from binary strings to binary strings) is uncountably infinite.

3. (11 points) Give the formal description of the machine M_4 in Figure 1.8 in the textbook.

4. (40 points) Draw state diagrams of deterministic finite automata that recognize the following languages. In all cases, the alphabet is {0,1}.
   a. {w | w begins with 0 and ends in 1}
   b. {w | the second symbol of w is 0 and $|w| \geq 4$}
   c. {w | w contains a single 00 and a single 11}
   d. {w | each 1 in w is immediately followed by a 0}
   e. {w | w does not contain 101 or 111}
   f. {w | w contains an odd number of 0s and at least two 1s}
   g. the set {ε}
   h. the empty set

5. (20 points) There are 53 students enrolled in this class. Assuming all (and only) enrolled students submit this assignment, prove that at least two of you will get the same number of total points for problems 3 and 4 combined. Then show that the other students in this class would not have been able to prove the same statement if you had followed that impulse on the first day to drop this class. (Thankfully, you didn't!). Assume that only integer-valued points are assigned between 0 and the maximum possible as indicated in the parentheses. [Hint: Use the pigeonhole principle]