CSE 322 Autumn 2004

Homework Assignment # 2

Due Date: Wednesday, October 20 (at the *beginning* of class)

1. (15 points) Seven homework assignments were assigned in a past offering of CSE 322. Six students each scored 100% in four of the assignments. Show that there was an assignment in which at least four students scored 100%. (Hint: Use the pigeonhole principle).

2. (15 points) We showed in class that the set Σ^* for $\Sigma = \{0,1\}$ is countably infinite. What is wrong with the following "proof" by diagonalization showing that Σ^* is uncountable?
   "Proof: By Contradiction. Suppose Σ^* is countably infinite. Then, there exists a bijection f: $N \rightarrow \Sigma^*$. We can create a new string s as follows:
   *i*th symbol of s = 0 if the *i*th symbol of f(*i*) is 1
   1 if the *i*th symbol of f(*i*) is 0
   1 if length of f(*i*) < *i* (i.e. *i*th symbol does not exist)
   Then, s differs from all the strings given by the function f. Since s is a binary string that is not among the outputs of f, this contradicts the fact that f is a bijection. Therefore, Σ^* is uncountable."

3. (20 points) Give the formal description of the finite automaton M_4 depicted in Figure 1.8 in the textbook.

4. (50 points) Draw state diagrams of (deterministic) finite automata that recognize the following languages. In all cases, the alphabet is {0,1}.
   a. {w | w begins with 00 and contains exactly three 1s}
   b. {w | the length of w is divisible by 3}
   c. {w | every 0 in w is followed by a 1}
   d. {w | w contains an even number of 0s and has odd length}
   e. {w | w contains at least two 0s and at most four 0s}
   f. {w | w begins with 0 and ends with 01} (this includes the string 01)
   g. {w |w does not contain the substring 00 or w has odd length}
   h. the set {ε}
   i. ∅
   j. the set of all strings except strings of length 2.