CSE 322
Winter Quarter 2003
Assignment 8
Due Friday, March 7, 2003

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) In this problem we explore the "top down" and "bottom up" construction for PDAs from context-free grammars. Consider the grammar $G = (V, \Sigma, R, E)$ where

$$\begin{aligned} V &= \{T, F, E\} \\ \Sigma &= \{+, *, (,), a\} \\ R &= \{E \to E + T, \\ &= E \to T, \\ &= T \to T * F, \\ &= T \to F, \\ &= F \to (E), \\ &= F \to a\} \end{aligned}$$

   (a) Design a PDA M_T by the "top down" construction that accepts $L(G)$. You may use a state diagram. Give a leftmost derivation of $(a + a) * a + a$. Beside it give the sequence of IDs from M_T that corresponds to the leftmost derivation.

   (b) Design a (extended) PDA M_B by the "bottom up" construction that accepts $L(G)$. You may use a state diagram. Give a rightmost derivation of $(a + a) * a + a$. Beside it give the sequence of IDs from M_B that corresponds to the rightmost derivation.

   The bottom up construction was given in class and is not found in the book. It works as follows. There is a state q which has two roles. The first role is to manage *reduce* steps. In a reduce step, if $A \to \alpha$ is a production, then the extended PDA in state q can remove α^R from the stack and replace it with A. The second role is to manage *shift* steps. In a shift step, the PDA in state q can take an input symbol and push it on to the stack. If SZ_0 ever appears on the stack then the PDA can move from q to its only accepting state p.

2. (10 points) In this problem we consider the conversion of context-free grammars to Chomsky normal form. Convert the following grammar to Chomsky normal form. Use the construction in class that has four steps: (i) Shorten the productions with long right hand sides, (ii) Remove the ϵ-productions, (iii) Remove the unit productions, (iv) For productions with right hands sides of length 2, remove those with terminals.

$$G = (V, \Sigma, R, S)$$

$$
\begin{aligned}
V &= \{S, A, B, C\} \\
\Sigma &= \{a, b\} \\
R &= \{S \rightarrow ABAB \mid A \\
&\quad\ A \rightarrow B \mid aaa \mid \epsilon \\
&\quad\ B \rightarrow abab \mid \epsilon\}
\end{aligned}
$$

3. (10 points) Consider the language $L = \{0^n 1 0^n 1 0^m : n \geq m \geq 0\}$. Use the pumping lemma for context-free languages to show that L is not context-free.