CSE 322: Formal Models in Computer Science
Sample Final Exam: Spring 2003

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DIRECTIONS:

- Closed Book, Closed Notes
- Time Limit: 1 hour 50 minutes
- Attempt all questions
- Maximum possible score = 200 points

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1. (40 points) Answer True or False to the following questions and briefly JUSTIFY each answer.

   (a) If a PDA M actually pushes a symbol on its stack then $L(M)$ is not regular.

   (b) For every infinite regular language L over Σ there are strings $x, y, z \in \Sigma^*$ such that $|y| \geq 1$ and for every $k \geq 0$, $xy^kz \in L$.

   (c) There is no algorithm to tell, given an arbitrary string P whether or not P is the ASCII for a syntactically correct C program.

   (d) If L is a CFL and $L = K \cap R$ for a regular language R then K is a CFL.

   (e) If L is not a CFL and $L = K \cap R$ for a regular language R then K is not a CFL.

   (f) There is no algorithm to tell, given an arbitrary program P and an input x, whether or not P runs forever on input x.

   (g) If the stack in a PDA M only has capacity for 100 characters on any input then $L(M)$ is regular.

   (h) If L is accepted by some PDA M then L^R is accepted by some PDA M'.

   (i) There is no algorithm to tell, given an arbitrary CFG G and input x whether or not $x \in L(G)$.

   (j) There is an algorithm to tell, given an arbitrary NFA N, whether or not $L(N) = \emptyset$.

2. (30 points) Classify each of the following sets as:

   A Both Regular and Context-Free,

   B Context-Free, but not regular,

   C Neither context-free nor regular but membership in the language can be decided by an algorithm.

   D Undecidable

   You do not need to justify your answers.

(a) $\{a^n b^m \mid m = 2n+1\}$. .... A B C D

(b) $\{a^n a^m \mid m = 2n+1\}$. .... A B C D

(c) $\{a^n b^m \mid m \equiv n \pmod 3\}$. .... A B C D

(d) $\{xcx^R \in \{a,b\}^* \mid x$ has an even number of a's$\}$. .... A B C D

(e) $\{a^i b^j c^k \mid k = i+j\}$. .... A B C D

(f) $\{a^i b^j a^k \mid k \neq i \text{ or } k \neq j\}$. .... A B C D

(g) $\{a^i b^j a^k \mid k \neq i \text{ and } k \neq j\}$. .... A B C D

(h) $\{a^n b^n a^m b^m \mid m, n \geq 0\}$. .... A B C D

(i) $\{a^n b^m a^n b^m \mid m, n \geq 0\}$. .... A B C D

(j) $\{xy \in \{a,b\}^* \mid x \neq y\}$. .... A B C D

3. (25 points)Let $L = \{0^n 1^m \mid m$ is an integer multiple of $n\}$.
Use the Myhill-Nerode theorem to show that L is not regular.

4. (30 points) Let $L = \{a^m b^n c^p \mid 0 \leq m < n < p\}$. Prove that L is not a context-free language.

5. (20 points) Let $G = (V, \Sigma, R, S)$ be the context-free grammar with the following set of rules:

$$S \rightarrow aSaSb \mid aSbSa \mid bSaSa \mid SS \mid e$$

   (a) Use one of the general constructions that convert a CFG to a PDA to give a PDA M such that $L(M) = L(G)$.

   (b) Show each step of a computation of your PDA (list the stack contents, state, and amount of input remaining) that accepts input $aabbaaaab$.

6. (25 points) Consider the grammar $S \rightarrow aS \mid aSbS \mid \epsilon$.

   (a) This grammar is ambiguous. Show in particular two parse trees and two leftmost derivations for the string aab.

   (b) What language does the grammar generate? (No justification necessary.)

   (c) Find an unambiguous grammar that generates the same language. (You don't have to prove unambiguity, but a one-sentence description of your main idea will help us understand your solution better.)

7. (30 points) For any context-free grammar $G = (V, \Sigma, R, S)$, we say that a nonterminal $A \in V$ is *useful* if there is a derivation $S \Rightarrow^* xAy \Rightarrow^* w$ for $w \in \Sigma^*$ and $x, y \in V^*$; otherwise it is *useless*. Suppose that G has the following rules:

$$S \rightarrow AC \mid BS \mid B$$

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\begin{array}{rcl}
A & \rightarrow & aA \mid aT \\
B & \rightarrow & CF \mid b \\
C & \rightarrow & cC \mid D \\
D & \rightarrow & aD \mid BD \mid C \\
E & \rightarrow & aA \mid BSA \\
T & \rightarrow & bB \mid b \\
U & \rightarrow & bA \mid a \mid Wb \\
W & \rightarrow & Ub \mid a \mid Bb
\end{array}
$$

(a) Which non-terminals of G are useful?

(b) Modify G to get an equivalent grammar G' whose nonterminals consist of precisely the nonterminals of G that are useful. (Don't remove any other non-terminals.)

(c) Describe a reasonably efficient algorithm that will remove all useless non-terminals from a grammar. Briefly argue (not formal proof necessary) why your algorithm works and thus show that any grammar is equivalent to one with only useful non-terminals.