CSE 322: Formal Methods in Computer Science Spring 2003

PROBLEM SET 7

Due Friday, May 29, 2003, in class

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1. (a) Show that if G is a context-free grammar in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n-1$ steps are required for any derivation of w.
   (b) Convert the grammar from Lewis and Papadimitriou, Problem 3.1.5 (the one that generates strings with an equal number of a's and b's), into Chomsky normal form.
2. Lewis and Papadimitriou, Problem 3.5.3.
3. Prove the following strengthening of the pumping lemma, where we require that **both** substrings v and y be nonempty when the string w is broken up as $uvxyz$: If L is a CFL, then there exists an integer $p \geq 1$ such that $\forall\ w \in L$, $|w| \geq p$, there exists a way to break down w as $w = uvxyz$, satisfying the conditions:
   (i) For each $i \geq 0$, $uv^ixy^iz \in L$
   (ii) $v \neq \epsilon$ **and** $y \neq \epsilon$
   (iii) $|vxy| \leq p$

   (the second condition is the stronger one compared to the version proved in class).
4. Use the pumping lemma for context-free languages to show that the following languages are not context-free. (You can use the version proved above if you wish.)
   (a) $L_1 = \{ww \mid w \in \{a,b\}^*\}$
   (b) $L_2 = \{a^ib^{i^2} \mid i \geq 1\}$
5. Give an example of a language that is **not** context-free and yet satisfies the conditions of the stronger version of the pumping lemma from Problem 3 above. (Note that you must *prove* that the language is not context-free, and if your construction is correct you obviously cannot use the pumping lemma to show this!)