

PROBLEM SET 4  
Due Friday, May 2, 2003, in class

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1. Lewis and Papadimitriou, Problem 2.4.7.

2. Define the language

$$L = \{a^i b^j c^k \mid i = 0 \text{ or } j = k\} .$$

(a) Show that  $L$  satisfies the requirements of the pumping lemma, namely show that there exists  $p \geq 1$  such that every  $w \in L$ ,  $|w| \geq p$ , can be rewritten as  $w = xyz$ ,  $|xy| \leq p$ ,  $y \neq \epsilon$ , such that  $xy^i z \in L$  for every  $i \geq 0$ .

(b) Is  $L$  regular? Why or why not? If not, why doesn't this contradict the pumping lemma? (Hint: Material from Section 2.5 on Myhill-Nerode theorem will be useful for this exercise.)

(c) (Extra credit) If you claimed that  $L$  is not regular in (b) above, can you generalize the pumping lemma so that it can demonstrate non-regularity of  $L$ ?

3. Lewis and Papadimitriou, Problem 2.4.1, Parts (a), (b), (c).

4. Use the pumping lemma to show that the following languages are not regular. Please structure and write your arguments as clearly as possible.

(a)  $L_1 = \{ww^R \mid w \in \{0, 1\}^*\}$

(b)  $L_2 = \{0^i 1^j \mid i > j \geq 0\}$

(c) (Extra credit) The language  $L_3$  consisting of strings of 0's and 1's, beginning with a 1, whose value treated as a binary number is prime. Eg.  $101 \in L_3$  since its value is 5, a prime number, while  $110 \notin L_3$  since its value is 6, a composite number.

5. For the top two DFAs on Page 61 of Lewis and Papadimitriou, find the minimum state equivalent DFA.