CSE 322: Formal Methods in Computer Science Spring 2003

PROBLEM SET 1

Due Friday, April 11, 2003, in class

Reminder: If you haven't done so already, subscribe to CSE 322 mailing list ASAP by visiting `http://majordomo.cs.washington.edu/mailman/listinfo/cse322`.

Instructions: Information on the collaboration policy and honor code in solving problem sets can be found off the course webpage — **please be sure to read it**. In a nutshell, you are allowed to collaborate with fellow students taking the class to the extent of discussing solution ideas, *provided you think about each problem on your own for at least 30 minutes.* You must write down solutions on your own, and must clearly ackowledge each person with whom you dicussed the solutions. You are expected to refrain from looking up solutions from websites or other literature.

Please be as clear as possible in your arguments and answers. Poorly written solutions, even if more or less correct, will be penalized.

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1. Consider the relation $R = \{(a,b),(b,c),(c,b),(c,d),(e,c)\}$ defined over the five element set $\{a,b,c,d,e\}$. What is the reflexive, transitive closure R^* of the relation R? Draw a drected graph representing R^*.

2. Let A, B, C be languages over an alphabet Σ.
   (a) Prove that $A \cdot (B \cup C) = A \cdot B \cup A \cdot C$.
   (b) Give an example of languages A, B, and C over some alphabet Σ such that $A \cdot (B \cap C)$ does not equal $A \cdot B \cap A \cdot C$.

3. Give state-diagrams for deterministic finite automata accepting each of the following languages over the alphabet $\{0,1\}$. Also, give a formal description of the DFA for the first of the following languages.
   (a) $L_1 = \{w \mid w \in \{0,1\}^*,\ w$ has an odd number of 0's and an even number of 1's $\}$
   (b) $L_2 = \{w \mid w \in \{0,1\}^*,\ w$ begins with 1, and which, interepreted as the binary representation of an integer, is divisible by $5\}$.
   (c) $L_3 = \{w \mid w \in \{0,1\}^*$, each 0 in w is immediately preceded by a $1\}$.

4. Describe informally the languages accepted by the last (bottom) three finite automata shown on page 61 of Lewis and Papadimitriou.

5. (Review of your induction proof skills. This problem is for your practice only, **No need to turn in solution for this.**)

   We usually write numbers in base 10. The language, L, of positive decimal numbers, is the set of strings over the alphabet $\Sigma = \{0,1,2,3,4,5,6,7,8,9\}$ that do not begin with 0. It is easy to see that L is exactly the set of strings generated by the following:

   - Any element of Σ other than 0 is a string in L.
   - If $x \in L$, then $xa \in L$ for every $a \in \Sigma$.

We can use this recursive definition to give recursive definitions of the value of a positive decimal number as well as its digit sum.

- For each $a \in \Sigma$, $\text{value}(a) = a$ and $\text{digitsum}(a) = a$.
- For each $x \in L$ and $a \in \Sigma$, $\text{value}(xa) = 10 \times \text{value}(x) + \text{value}(a)$ and $\text{digitsum}(xa) = \text{digitsum}(x) + \text{digitsum}(a)$.

Now to your exercises:

(a) Give a deterministic finite automaton that recognizes L.

(b) Use modular arithmetic and induction on the definitions of value and digitsum to prove the 'Rule of 3' which says that a number is divisible by 3 if and only if its decimal digits sum to a multiple of 3.