CSE 322
Autumn Quarter 2003
Assignment 2
Due Friday, October 10, 2003

All solutions should be neatly written or type set. All major steps in proofs must be justified.

1. (10 points) This problem is designed to strengthen your ability to prove facts by induction. The reversal w^R of a string w can be defined recursively in the following way.

$$\begin{aligned} \varepsilon^R &= \varepsilon \\ (xa)^R &= ax^R \end{aligned}$$

   where $a \in \Sigma$.

   Prove the following: For all strings x and y over Σ, $(xy)^R = y^R x^R$. For this your proof should be by induction on the length of y. You may use recursive definition of reversal and any basic facts about concatenations such as associativity and the identity properties of ε.

2. (10 points) Design deterministic finite automata using a state transition diagram for each of the following languages.

   (a) $\{x \in \{0,1\}^* \ : \ 101$ is a substring of $x\}$.

   (b) $\{x \in \{0,1\}^* \ : \ 111$ is not a substring of $x\}$.

   (c) $\{x \in \{0,1\}^* \ : \ x$ contains exactly 5 $0's\}$.

   (d) $\{x \in \{0,1\}^* \ : \ x$ has an odd number of $0's$ or an even number of $1's\}$.

3. (10 points) Consider the languages

$$L_k = \{x \in \{0,1\}^* \ : \ x \text{ contains exactly } k \ 0's\}$$

   for $k \geq 0$.

(a) Formally define a deterministic finite automaton M_k with exactly $k + 2$ states that accepts L_k.

(b) Prove by contradiction that every deterministic finite automaton that accepts L_k has at least $k + 2$ states. The ideas from problem 1 of assignment 1 are useful.