

CSE 322
Autumn Quarter 2003
Assignment 1
Due Friday, October 3, 2003

All solutions should be neatly written or type set. All major steps in proofs must be justified.

1. (5 points) This problem involves an application of the Pigeon Hole Principle. Simply stated the Pigeon Hole Principle states that if there are m items placed in n bins and $m > n$, then some bin contains at least 2 items.

Suppose we have a directed graph $G = (V, E)$ where V has n vertices. A path of length m in G is a sequence of vertices v_0, v_1, \dots, v_m where (v_i, v_{i+1}) is an edge for $0 \leq i < m$. The vertices v_0, v_1, \dots, v_m are *visited* by the path. Note that the number of edges on the path is its length. Use the Pigeon Hole Principle to prove that any path in G of length $\geq n$ visits some vertex at least twice.

2. (5 points) This problem involves a practice doing an induction proof.

Suppose we have an alphabet of $m > 0$ symbols. Show by induction that the number of strings of length n in these symbols is m^n . Note that there is only one string of length 0, namely the empty string.

3. (5 points) This problem involves doing a proof by contradiction.

Prove that the $\sqrt{3}$ is not a rational number. Recall that rational numbers are those that can be represented by fractions.