CSE 322: Regular Expressions and Finite Automata

- **Last Time**: Definition of a Regular Expression
  - R is a regular expression iff
    R is a string over $\Sigma \cup \{ \varepsilon, \varnothing, (,), \cup, * \}$ and R is:
    1. Some symbol $a \in \Sigma$, or
    2. ε, or
    3. $\varnothing$, or
    4. $(R_1 \cup R_2)$ where R_1 and R_2 are regular exps., or
    5. $R_1R_2 = R_1 \circ R_2$ where R_1 and R_2 are reg. exps., or
    6. R_1^* where R_1 is a regular expression.
- **Precedence**: Evaluate * first, then $\circ$, then $\cup$
  - E.g. $0 \cup 11^* = 0 \cup (1 \circ (1^*)) = \{0\} \cup \{1, 11, 111, \ldots\}$

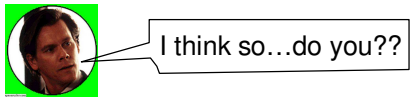
Examples

- What is R for each of the following languages?
  1. L(R) = {w | w contains exactly two 0's}
  2. L(R) = {w | w contains at least two 0's}
  3. L(R) = {w | w contains an even number of 0's}
  4. L(R) = {w | w is a valid identifier in C}
  5. L(R) = {w | w is what you hear in a Kevin Bacon movie}
  6. L(R) = {w | w does not contain 00}

(Example on board)

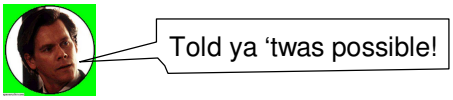
Regular Expressions and Finite Automata

- What is the relationship between regular expressions and DFAs/NFAs?
- Specifically:
  1. **R NFA**? Given a reg. exp. R, can we create an NFA N such that L(R) = L(N)?
  2. **NFA R**? Given an NFA N (or its equivalent DFA M), can we come up with a reg. exp. R such that L(M) = L(R)?



From Regular Expressions to NFAs

- Problem: Given *any* regular expression R, how do we construct an NFA N such that L(N) = L(R)?
- Soln.: Use the multi-part definition of regular expressions!!
  - Show how to construct an NFA for each possible case in the definition: R = *a*, or R = ε, or R = ∅, or R = (R1 ∪ R2), or R = R1°R2, or R = R1*.



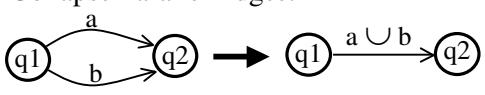
- Example: Draw NFA for 01Σ*10

From NFAs/DFAs to Regular Expressions

- Problem: Given *any* NFA N, how do we construct a regular expression R such that L(N) = L(R)?
- Solution: First, convert NFA N to an equivalent DFA M to keep things simple. Then:
  - **Idea**: Collapse 2 or more edges in M labeled with single symbols to a *new edge* labeled with an *equivalent regular expression*
  - This results in a **"generalized" NFA** (**GNFA**)
  - **Our goal**: Get a GNFA with 2 states (start and accept) connected by a single edge labeled with the required regular expression R

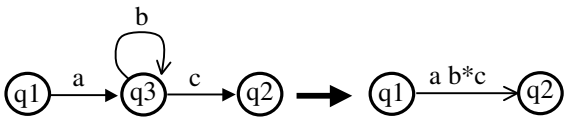
From NFAs/DFAs to Regular Expressions

- Steps for extracting regular expressions from DFAs:
  1. Add new start state connected to old one via an ε–transition
  2. Add new accept state receiving ε–transitions from all old ones
  3. Keep applying 2 rules until only start and accept states remain:
     1. Collapse Parallel Edges:



Note: Also applies to q1 = q2

     2. Remove "loopy" states:



Note: Also applies to q1 = q2

(Example: On board and in textbook)