CSE 322
Winter Quarter 2001
Assignment 7
Due Friday, February 23, 2001

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. (10 points) Design a context-free grammar that generates the set of regular expressions (fully parenthesized) over an alphabet Σ. In your definition you will want to differentiate the symbol ε from the empty string.

2. (10 points) Given context-free grammars $G_1 = (V_1, \Sigma_1, R_1, S_1)$ and $G_2 = (V_2, \Sigma_2, R_2, S_2)$, design context-free grammars G such that:

   (a) $L(G) = L(G_1)L(G_2)$ (concatenation),

   (b) $L(G) = L(G_1)^*$ (Kleene star),

   (c) $L(G) = L(G_1)^R$ (reversal).

3. (10 points) Consider the context free-grammar:

$$\begin{array}{rcl} S & \rightarrow & ASA|A|\varepsilon \\ A & \rightarrow & 011|\varepsilon \end{array}$$

   Use the method described in class to convert the grammar into Chomsky normal form. In this method do the steps in the following order: (i) add a new start symbol if ε is generated by the grammar, (ii) shorten productions whose right hand sides are longer than 2, (iii) remove ε-rules, (iv) remove unit rules, (v) make all right hand sides of length 2 into nonterminals.

4. (10 points) A context-free grammar $G = (V, \Sigma, R, S)$ is *right-linear* if every production in R has one of the following forms (i) $A \rightarrow wB$ where $A, B \in V$ and $w \in \Sigma^*$ or (ii) $A \rightarrow w$ where $A \in V$ and $w \in \Sigma^*$.

   (a) Consider the right-linear grammar $G = (V, \Sigma, R, S)$ where

$$\begin{array}{rcl} V & = & \{S, A, B\} \\ \Sigma & = & \{a, b\} \\ R & = & \{S \rightarrow aaS, S \rightarrow abA, S \rightarrow \varepsilon, A \rightarrow bA, A \rightarrow baB, \\ & & B \rightarrow aB, B \rightarrow bbS\} \end{array}$$

   Design an NFA that accepts $L(G)$.

   (b) Prove that the language generated by a right-linear grammar is always regular.