CSE 322
Winter Quarter 2001
Assignment 6
Due Friday, February 16, 2001

All solutions should be neatly written or type set. All major steps in proofs and algorithms must be justified.

1. In this problem you will explore how to convert an ambiguous context-free grammar into an unabibuous one. A context-free grammar is ambiguous if there is some $w \in L(G)$ such that w has at least two distinct parse tree (or equivalently, w has at least two different leftmost derivations). Consider the context-free grammar $G = (\{S, a, b\}, \{a, b\}, R, S)$ where

$$R = \{S \to SS, S \to aSb, S \to ab\}.$$

   (a) Demonstrate that G is ambiguous by showing two distinct parse trees for some string in $L(G)$

   (b) Give an alternative grammar for $L(G)$ that is not ambiguous.

2. In this problem you will design a context-free grammar. Design a context-free grammars for the language

$$L = \{a^n b^m c^{n+m} : n, m \geq 0\}$$

   and give a derivation of *abbccc*. For each nonterminal in your grammar explain what set of strings is derived by the nonterminal.

3. In this problem you will explore some closure properties of context-free languages.

   (a) Show that the context-free languages are closed under union. That is, given two context free grammars $G_1 = (V_1, \Sigma_1, R_1, S_1)$ and $G_2 = (V_2, \Sigma_2, R_2, S_2)$ construct a new grammar G such that $L(G) = L(G_1) \cup L(G_2)$.

   (b) Show that the context-free languages are not closed under intersection. You can use the fact that the language $\{0^n 1^n 2^n : n \geq 0\}$ is *not* context-free.

   (c) Show that the context-free languages are not closed under complement.