University of Washington
Department of Computer Science and Engineering
CSE 321, Winter 1998

February 2, 1999

Midterm Exam, February 11, 1998

NAME: ______________________

Instructions:

- Closed book, closed notes.
- Time limit: 50 minutes.
- Answer the problems on the exam paper.
- If you need extra space use the back of a page.
- Problems are not of equal difficulty, if you get stuck on a problem, move on.

1	/20
2	/20
3	/10
4	/20
5	/20
6	/10
7	/20
Total	/100

Problem 1 (20 points):

a) Show that the expression $(p \rightarrow q) \rightarrow r$ is a contingency.

b) Give an expression using the logical operators $\neg$, $\vee$ and $\wedge$ which is logically equivalent to $(p \rightarrow q) \rightarrow r$.

Problem 2 (20 points):

Suppose $g_0 = 1$, $g_1 = 2$ and $g_n = g_{n-1} \cdot g_{n-2}$ for $n \geq 2$.

Prove that $g_n = 2^{f_n}$, where $f_0, f_1, f_2, \ldots$ are the Fibonacci numbers.

(Recall: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$.)

Problem 3 (10 points):

Prove or disprove: All odd Fibonacci numbers greater than 1 are prime.

Problem 4 (20 points):
Consider the set S of strings over the alphabet $\Sigma = \{a, b\}$ defined by the following rules:

a) $\lambda \in S$.

b) If $w \in S$, then $awbb \in S$.

c) If $x \in S$, $y \in S$, and $z \in S$, then $xyz \in S$.

We also assume that nothing belongs to S unless it can be generated by the above rules.

a) List all elements of S of length less than 10.

b) Prove that all elements of S have at least as many b's as a's.

Problem 5 (20 points):
How many four digit numbers:

a) Are made up of distinct digits (e.g., 1407)

b) Are made up of distinct digits in increasing order (e.g., 0147)

c) Start and end with the same digit (e.g., 3983)

d) Contain exactly two distinct digits (e.g., 9399)

You do not need to multiply out binomial coefficients or factorials.

Problem 6 (10 points):
Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9.

Problem 7 (20 points):
Using the predicates:

$Likes(p, f)$: "person p likes to eat f."
$Serves(r, f)$: "restaurant r serves f."

translate the following statements into logical expressions:

a) Every restaurant serves a dish which Bart likes, but Lisa does not like.

b) There is a restaurant which only serves food liked by everyone.

Translate the following logical expressions into English. (You may want to give a couples of sentences of explanation – the point of this question is to demonstrate that you understand the logical expression.)

c) $\exists r \forall p \exists f (serves(r, f) \wedge likes(p, f))$

d) $\forall r \exists p \forall f (serves(r, f) \wedge likes(p, f))$