University of Washington March 8, 1999
Department of Computer Science and Engineering
CSE 321, Winter 1998

Final Exam, March 17, 1998

Problem 1 (20 points):
Give *short* definitions for the following terms. Do not write more than two sentences for each answer.

a) Total Order

b) Equivalence Relation

c) Hamiltonian Circuit

d) Subgraph

e) Bipartite Graph

f) Symmetric Closure

g) Product Rule

h) Rule of Inference

i) Predicate

j) Conditional Probability

Problem 2 (10 points):
What is the minimum number of colors required to color K_n. Justify your answer.

Problem 3 (10 points):
Answer the following questions about K_7. (K_7 is the complete graph on 7 vertices.) Justify your answers.

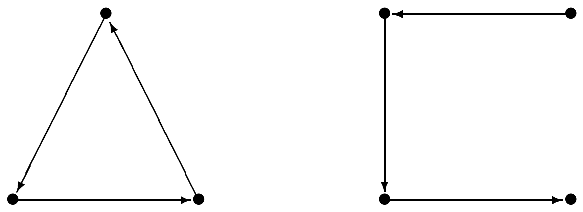
a) Does K_7 have a Hamiltonian circuit?

b) Does K_7 have an Eulerian tour?

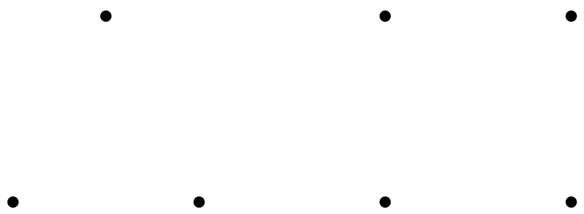
c) Is K_7 planar?

Problem 4 (10 points):

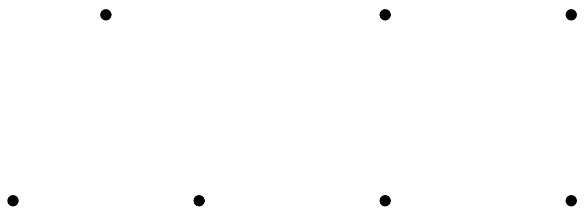
Consider the relation R defined by the following directed graph:



a) (10 points) Draw the relation R^7.



b) (10 points) Draw the transitive closure of R.



Problem 5 (10 points):

Let G_n be the set of all graphs on n vertices. Define the relation R as $(g_1, g_2) \in R$ if g_1 is isomorphic to g_2. Is R an equivalence relation? Justify your answer.

Problem 6 (10 points):

Let the set S be defined by

a) $5 \in S$ and $9 \in S$.

b) if $x \in S$ and $y \in S$ then $x + y \in S$.

Prove using strong induction that all integers greater than or equal to 32 are in S.

Problem 7 (10 points):

Suppose that A and B are square matrices with the property $AB = BA$. Show that $AB^n = B^n A$ for every positive integer n.

Problem 8 (10 points):

Let $\gg$ be an order relation with domain A. Recall that an element x is *maximum* if x is greater than all other elements, and x is *maximal* if there are no other elements greater than x.

Translate the following statements into logical expressions. You may use the following predicates: "$u \gg v$", "$u \not\gg v$", "$u = v$", "$u \neq v$".

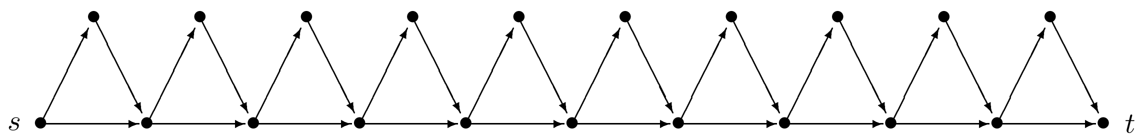
a) x is a maximum element.

b) x is a maximal element.

c) The relation $\gg$ has no maximal elements.

Problem 9 (10 points):

How many paths are there between s and t of length k, $(10 \leq k \leq 20)$. Give your answer as a formula involving k. (Do not use matrix multiplication to count the paths.)



Problem 10 (10 points):

Suppose that $P_1, P_2, \ldots, P_n$ are independently assigned random truth values with

$$\text{Prob}[P_k \text{ is true}] = \frac{1}{k}.$$

What is the probability that $P_1 \oplus P_2 \oplus \cdots \oplus P_n$ is true? Justify your answer.

Hint: Figure out the answer for small values of n (1, 2, 3, and 4), and then solve the general case.