University of Washington
Department of Computer Science and Engineering
CSE 321, Fall 1994
Handout No. 16

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Final Exam questions from f89, f91, s92

These are final exam questions from previous offerings for the course. Since lecture topics vary from year to year, some of the questions relate to material that was not presented this year.
The fall 94 final will cover *all* the material presented in class: 1.1-1.3, 3.1-3.3, 4.1-4.5, 6.1-6.6, 7.1-7.5, 7.7-7.8. Approximately 1/3 of the exam will be pre-midterm (1.1-4.2), and 2/3 of the exam will be post-midterm (4.3-7.8).

Fall 89:

Problem 1:

Show that the proposition

$$p \rightarrow ((q \rightarrow (r \rightarrow s)) \rightarrow t)$$

is a contingency *without* constructing its full truth table.

Problem 2:

Using the universe of integers for all quantifiers express the following English sentences in predicate logic using the predicates, P, E, and L where $P(x)$ means 'x is prime', $E(x)$ means 'x is even', and $L(x,y)$ means '$x < y$'.

(a) No prime greater than 2 is even.

(b) For every number, there is a larger prime number.

(c) Translate the following predicate logic expression into English where the predicates are the same as above:

$$\forall x[(E(x) \wedge P(x)) \rightarrow L(x, 100)].$$

Problem 3:

In an undirected graph G, we define $dist(v,w)$ to be the length of the shortest path from v to w. Show that for vertices x, y, z, $dist(x,z) \leq dist(x,y) + dist(y,z)$.

Problem 4:

Let $G = (V, E)$ be a directed graph. Define a relation R on E by $(e_1, e_2) \in R$ iff e_1 and e_2 lie on a common simple circuit. (Recall that a simple circuit is a path that starts and ends at the same vertex, and does not repeat any edges).

(a) Is R necessarily reflexive? Prove or disprove.

(b) Is R necessarily symmetric? Prove or disprove.

(c) Is R necessarily transitive? Prove or disprove.

Problem 5:

Prove that an undirected tree can be colored with two colors such that if (v, w) is an edge in the tree, v and w receive different colors.

Problem 6:

Does there exist a full 10-ary tree with exactly 1990 vertices? Justify your answer. MATERIAL FOR THIS PROBLEM WAS NOT COVERED FALL 1994

Problem 7:

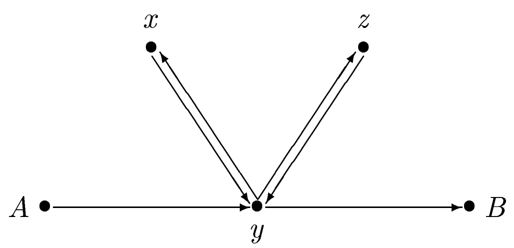
What is the cardinality of the set of all PASCAL programs? Justify your answer. MATERIAL FOR THIS PROBLEM WAS NOT COVERED FALL 1994

Problem 8:

Let R be a relation on a set A. Is the transitive closure of R always equal to the transitive closure of R^2? Prove or disprove.

Problem 9:

Prove that in the graph below, there are exactly 2^{n-1} paths of length $2n$ between vertex A and vertex B for $n \geq 1$.



Fall 91:

Problem 1:

The formula

$$\neg(p \to \neg q) \to (r \to (\neg s \to t))$$

is false for exactly one assignment to p, q, r, s and t. Find this assignment *without* constructing a truth table.

Problem 2:

Find predicates $P(x)$ and $Q(x)$ such that $\forall x(P(x) \vee Q(x))$ is true, but $\forall x P(x) \vee \forall x Q(x)$ is false. Justify your answer.

Problem 3:

Determine the number of strings over $\{a, b, c\}$ of length 100 that have exactly 98 a's.

Problem 4:

Suppose that you are working on a True-False test with 10 questions. Let p_i be the probability that you get answer i correct. Assume that the probabilities are independent.

(a) What is the probability of getting all of the answers wrong (expressed as a formula involving $p_1, p_2, \ldots, p_{10}$).

(b) If $p_i = 1 - 2^{-i}$, what is your expected number of correct answers.

Problem 5:

Give an example of a relation which is not reflexive, not symmetric, not anti-symmetric, and not transitive. (You are to give one relation that lacks all of these properties, not separate relations for each property.) Justify your answer.

Problem 6:

Give the graph representation and the matrix representation of the transitive closure of the relation $\{(a, b), (b, c), (c, a), (d, d)\}$. The domain of the relation is $\{a, b, c, d\}$.

Problem 7:

Give three different equivalence relations on the set $\{a, b, c, d\}$.

Problem 8:

Let the relation $\ll$ be defined on ordered pairs of positive integers by

$$(a, b) \ll (c, d) \text{ if } ab < cd \text{ or } (a = c \text{ and } b = d).$$

Show that $\ll$ is a partial order, but $\ll$ is not a total order.

Problem 9:

Let G be a directed graph with n vertices. Show that if G has a path of length greater than n, then G has a cycle.

Problem 10:

Give a pair of non-isomorphic undirected graphs each with six vertices where every vertex has degree exactly two.

Problem 11:

Let $G = (V, E)$ be an undirected graph. The complement of G, $\overline{G}$ is defined as the graph $\overline{G} = (V, \overline{E})$ where $\overline{E} = \{(v, w) \mid v \in V, w \in V, v \neq w, (v, w) \notin E\}$.
Give an example of a graph G such that both G and $\overline{G}$ contain Hamiltonian circuits.

Problem 12:

Let $G = (V, E)$ be a graph with $|V| \geq 5$. Prove that if G is 2-colorable (bipartite), then $\overline{G}$ is not 2-colorable.

Spring 92:

Problem 1:

The following terms are used in the propositional calculus. Give a one sentence definition for each of the terms.

a) Contingency

b) Contrapositive

c) Converse

Problem 2:

Give logical expressions for the following statements. Use quantifiers, connectives, and the predicates $P(x)$ and $H(x)$ which mean "x passed the class" and "x turned in all of the homework".

a) Every student that passed the class turned in all of the homework.

b) There was a student that passed the class, but did not turn in all of the homework.

Problem 3:

Consider the English alphabet, which consists of 5 vowels and 21 consonants.

a) How many strings of length 10 consist of 5 vowels followed by 5 consonants?

b) How many strings of length 10 consist of 5 distinct vowels followed by 5 distinct consonants. (In other words, the string has no repeated characters.)

You may express your answers as products of integers.

Problem 4:

Suppose that propositions $p_1, p_2, \ldots, p_{n+1}$ are independently assigned random truth values, where the probability that p_i is true is $\frac{1}{2}$.

a) What is the probability that the compound proposition $p_1 \wedge p_2 \wedge \cdots \wedge p_n$ is true?

b) What is the probability that the compound proposition $(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow p_{n+1}$ is true?

Problem 5:

Let $A = \{x, y, z\}$, $B = \{1, 2, 3, 4\}$, and $C = \{a, b, c\}$. Let R be the following relation from A to B:

$$R = \{(x, 1), (x, 2), (y, 2), (y, 3), (z, 3)\},$$

and S be the following relation from B to C:

$$S = \{(1, a), (2, a), (2, b), (3, b), (4, b), (4, c)\}.$$

Compute the composition of R and S.

Problem 6:

Suppose R_1 and R_2 are transitive relations on a set A. Is the relation $R_1 \cup R_2$ necessarily a transitive relation? Justify your answer.

Problem 7:

Suppose R is the relation on the integers where xRy if and only if $x = y + 1$. Describe the relation that is the transitive closure of R.

Problem 8:

Let the relation $\ll$ be defined on the integers by

$$x \ll y \text{ if } |x| < |y| \text{ or } (|x| = |y| \text{ and } x \leq y).$$

($|y|$ denotes the absolute value of y.)

a) Lists the elements of $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ under the $\ll$ ordering.

b) Argue that this ordering relation is total.

Problem 9:

For which values of n does K_n contain an Eulerian circuit. (K_n denotes the complete graph on n vertices, K_n is undirected.) Justify your answer.

Problem 10:

Let $G = (V, E)$ be the undirected graph that has as its vertices the set of integers, and edges: $E = \{(x, x + 3) \mid x \text{ is an integer}\}$.
Describe the connected components of G.

Problem 11:

How many edges do the following undirected graphs contain.

a) K_n, the complete graph on n vertices.

b) $K_{n,m}$, the complete bipartite graph between n vertices and m vertices.

c) Q_n, the n dimensional hypercube. (Q_n has 2^n vertices.)