University of Washington
Department of Computer Science and Engineering
CSE 321, Fall 1994
Handout No. 17

March 6, 1998

Final Exam, December 13, 1994

Problem 1 (50 points):

Give *short* definitions for the following terms. Do not write more than two sentences for each answer.

a) Euler's formula

b) Eulerian Circuit

c) $K_{n,m}$

d) Toplogical Sort

e) Equivalence Relation

f) Lexicographic Order

g) Composition of Relations

h) Random Variable

i) Binomial Coefficient

j) Conditional Probability

Problem 2 (20 points):

For what values of n does the n-dimensional hypercube (Q_n) contain an Eulerian Circuit? Justify your answer.

Problem 3 (20 points):

Let $G = (V, E)$ be an undirected graph. The complement of G, $\overline{G}$ is defined as the graph $\overline{G} = (V, \overline{E})$ where $\overline{E} = \{(v, w) \mid v \in V,\ w \in V,\ v \neq w,\ (v, w) \notin E\}$.

a) (5 points) Recall that C_5 is the cycle of length 5. Draw C_5 and $\overline{C_5}$.

b) (15 points) Show that C_5 is isomorphic to $\overline{C_5}$.

Problem 4 (20 points):

Give the Hasse diagrams for four different ordering relations on the set $\{a, b, c, d\}$.

Problem 5 (20 points):

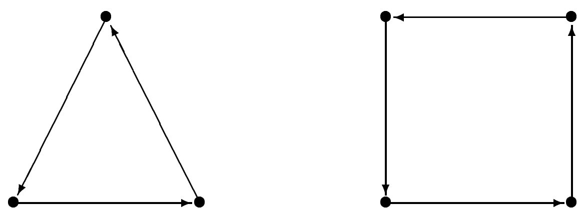
Define a relation $x \gg y$ on the positive integers by: $x \gg y$ if $x = y$ or if there exists a $k > 0$ such that $2^k \mid x$ but $2^k \nmid y$.
For example, $12 \gg 18$ since $4 \mid 12$ but $4 \nmid 18$.

a (10 points) Prove that $\gg$ is a partial order, but not a total order.

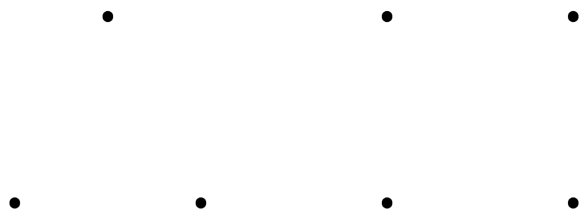
b (10 points) Describe a total order on the positive integers which is compatible with $\gg$.

Problem 6 (20 points):

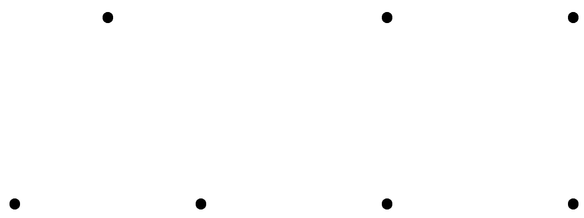
Consider the relation R defined by the following directed graph:



a) (10 points) Draw the relation R^6.



b) (10 points) Draw the transitive closure of R.



Problem 7 (20 points):

Suppose that you are taking a true-false test that contains the statements P and Q. You do not have a clue as to the truth values of P and Q, but you do know that P implies Q. Give a strategy that ensures that you get at least one of the answers correct and explain why your strategy works.

Problem 8 (20 points):

Give an induction proof to show that for all $n \geq 1$:

$$\sum_{j=1}^{n}(2j-1) = n^2.$$

Problem 9 (20 points):

Let $t(R)$ denote the transitive closure of R. Give an example of relations R_1 and R_2 over the set $A = \{a, b, c, d\}$ such that

$$t(R_1 \cup R_2) \neq t(R_1) \cup t(R_2).$$

Justify your answer.

Problem 10 (20 points):

A computer system requires passwords of length 6, each of which must contain exactly 3 digits (from 0 to 9) and exactly 3 lower case letters. Characters may be repeated, and the letters and digits may occur in any order, so "a37xa9" is a valid password.

What is the total number of valid passwords? You may express your answer as a formula involving binomial coefficients.