Discrete Structures — Anna Karlin, 426C Sieg
CSE 321, Spring 1998

Homework #5

Due at the beginning of class, Wednesday, May 13

Reading: Rosen, Sections 4.1 – 4.4

New Experimental Policy: You are strongly encouraged to work in groups of 2 on this homework and turn in a single homework with both names on it. Both members of the group will receive the same score.

1. Consider the following variant of the stable pairing problem, which we'll call the "general stable pairing problem" (this is the bisexual version): We are given $2n$ people, each with a complete ranking of the remaining $2n-1$ people (no ties). A "pairing" of the $2n$ people is a partition of the $2n$ people into n couples. A pairing is "unstable" if there are two people (not a couple) that both prefer each other to the people with whom they are coupled. A pairing is "stable" if it it not unstable. Prove that there exist preference lists for which there is no stable pairing (i.e., disprove the statement that for any set of preference lists given as input to the general stable pairing problem, there is at least one stable pairing). Hint: study the example on page 4 of the notes.

2. Prove that the Traditional Marriage Algorithm must terminate on the day that every girl has received a proposal.

3. In class we argued that the Traditional Marriage Algorithm will never require more than n^2 days to terminate. In fact, it is possible to prove a tighter upper bound, $n^2 - 2n + 2$, on the maximum number of days until the algorithm terminates.

   - Describe a set of preference lists that requires $n^2 - 2n + 2$ days to terminate.
   - **Extra Credit.** Prove that for any set of preference lists, it can take no longer than n^2-2n+2 days until the algorithm terminates.

4. Rosen, Section 3.3, Problem 6.

5. Rosen, Section 3.3, Problem 12.

6. Rosen, Section 4.1, problem 16.

7. Rosen, Section 4.1, problem 34.